The choices in theoretical physics from Galilei to Einstein

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Abstract: The history of the emergence of two dichotomies in the foundations of theoretical physics is illustrated. They have generated ethical choices, which have been taken by theoretical physicists. Their implications for both philosophy of science and ethics are derived.

Keywords: Infinity, organization, dichotomy, Newton's paradigm, Einstein, Leibniz' labyrinths, philosophy of sciences, ethics.

1. The dilemma on the kind of infinity during the birth of modern science

Koyré's history of the birth of modern science (Koyré 1957) has stressed several choices. Galilei's reflections on this subject are very interesting. In the first day of the *Discourses on Two New Sciences*, Galilei deals with the problems of the kind of infinity, the degrees of infinity and the indivisibles. He discusses for a long time the significant differences between actual infinity and potential infinity. He tackles the question whether and how to choose between the two kinds of infinity, a question which appears at the same time of a philosophical nature and a scientific nature. Galilei (1954) stresses the possible choice between the two kinds of infinity; yet, he does not see any decision criterion, although he well knows that his disciple, Cavalieri, by choosing the actual infinity, has invented a new calculus.

2. The choice of the actual infinity established by Newton's mechanics. A theoretical monopoly on theoretical physics

Various were the choices taken by other scientists. Descartes rejected the actual infinity and chose the "undefined" (i.e. potential infinity), although his geometric optics treats the points at infinity of a beam of light as the usual points opposed (i.e. actual infinity). Also Huygens refused to solve problems by means of the infinitesimals (which were considered as the inverse numbers of the actual infinity); he consistently tried to prove his theorems (e.g. the brachistochrone problem) by means of a calculus of finite quantities. Although he invented infinitesimal analysis by means of the actual infinity, subsequently Leibniz wanted to found theoretical physics on the potential infinity only (Drago 2003).

The choices of the above scientists are commonly interpreted as contingent events of the time elapsed before the consolidation of theoretical physics, which occurred through the birth of Newton's mechanics. Without explanations, Newton chose the actual infinity in theoretical physics. This fact was an essential element of what in subsequent times has constituted the Newtonian paradigm.

Moreover, Newton has interpreted a beam light as the trajectory of massive particles; the laws of reflection and refraction (as well as interference) have been obtained by him as a consequence of his mechanical laws (Newton 1704). In such a way he was successful in reducing geometrical optics to a particular case of his mechanics' theory. As a consequence, his mechanics was a global theory, a monolith; no different physical theory could exist. Hence, after Newton's mechanics, theoretical physics excluded any choice. After this theoretical experience, *two different theories* on the same set of phenomena have been considered according to two relationships, i.e. either *mutually contradictory* or *including one into the other*.

In addition, Newton has extended the monopoly of theoretical physics to the entire human rational thinking. In the last part of his last book, *Optiks*, he listed 31 Queries, which in the subsequent times have represented all the problems to be solved through his theory. These problems range from the chemical problem to even - in the Query 31 - a new foundation of the ethics of "the ancient cardinal virtues" through the mechanical laws governing the heavens. Hence, Newton planned his theory as an omniscient science, ethics included as a particular case.

3. Theories emerging outside Newton's paradigm. The characterization of different formulations as merely technical variants

Some new physical principles have been proposed; e.g. by Fermat in optics, Maupertuis in mechanics, etc. Each of these principles was presented in metaphysical terms, rather than in physical terms only; moreover, each of them was not developed so much to become a completed theory. For these reasons they did not constitute an alternative to Newton's mechanics.

In 1717 the mathematical formula of a further principle, that of virtual work (PVW), was suggested. Yet, it seemed no more than a practical rule for engineers' use. Only some isolated theorists (the Bernoulli's) have appreciated it. In 1754 D'Alembert suggested a non-metaphysical principle, but its meaning was obscure and it was obscurely applied by him. It was overlooked also because it mainly concerned the impact of bodies, i.e. discrete phenomena, instead of continuous phenomena, which at that time were privileged in order not only to follow Newton's choice for the actual infinity, but also to explain the continuous motions of heaven's bodies. Hence, theoretical physicists did not perceive any competition between these new principles and Newton's mechanics.

However, around the years of the French Revolution some surprising theories born. Two well-formulated theories of mechanics were founded by Lazare Carnot (1783) and Lagrange (1788). The former theory did not rely on calculus but only on an elementary mathematics (i.e. trigonometry and vector calculus); moreover, it does not include idealist notions (L. Carnot 1803, p. 3; Dugas 1950, p. 309). In addition, by maintaining that even the mathematical notions have to be linked to the empirical data, Lazare

Carnot has criticized the idealistic notions of Newton's mechanics (e.g. absolute space). Thus, his choice was for potential infinity. According to a pluralist attitude about this dichotomy on the kind of infinity, he considered as legitimate also the alternative choice of the use of infinitesimals. Yet, subsequently Carnot's formulation was characterized as a theory about mechanical machines, without relevance for theoretical physics. Lagrange deliberately has based his mechanics' theory on the infinitesimals (Lagrange 1788, p. ii) notwithstanding he had previously invented an "algebraic" foundation of calculus (Lagrange 1797).

Surprisingly, these two new formulations were not in mutual contradiction; nor were they in contradiction with Newton's theory. Their relationships have been discussed through their basic principles. Lagrange based its formulation on the PVW because he stressed that Newton's principles couldn't describe constraints. He obtained a more powerful formulation; its results were so impressive that the PVW was received as a basic principle of mechanics. The question arose of the relationship between this new principle and Newton's principles. By wanting to re-establish Newton's theory as the unique foundation of mechanics, the most celebrated scientists have attempted to deduce the PVW from Newton's principles (usually through the lever's law or the laws of some more sophisticated mechanical tools). A great debate on the role played by the PVW in theoretical physics born. Yet this debate was eventually inconclusive (Poinsot 1975; Drago 1993; Capecchi, Drago 2005).

Rather, after Cauchy (and then Weierstrass) reformed infinitesimal analysis by expelling infinitesimals, the theoretical physicists considered this progressive and assured mathematical theory as the best basis for re-structuring mechanics. As a result, a "rational mechanics" was proposed – as a present day all textbooks of rational mechanics show – as the unifying framework for the entire theoretical mechanics, including Lagrange mechanics and all formulations derived by the other principles. After this trust in the power of the calculus, at the end of the XIX century the prevailing opinion on the different formulations of mechanics was to consider their differences as merely technical in nature. Even Mach agreed, although he launched a program for building an alternative to Newton's mechanics (Mach 1883, chapter VIII).[1] The above-mentioned appraisal of a technical equivalence of all formulations re-established a monist view on science. It again excluded any choice, yet at the cost of assuming a lot of philosophical prejudices on the divergent physical theories – as we will see in the following.

During and after the French revolution new physical theories, irreducible to the Newtonian paradigm, born. Chemistry was a completely new scientific theory; yet, it was considered as an "art" owing to its apparent lack of the infinitesimal analysis. New optical phenomena (essentially, diffraction) originated the physical optics. As a consequence, the mathematics of Optics changed from the elementary notions of Euclidean geometry to the most advanced calculus, i.e., partial differential equations. In correspondence, the role played by this theory changed: no longer a subordinate theory,

[1] Yet Mach advanced the hypothesis that the continuum is maybe an appearance only, being all discrete. He elaborated this hypothesis on the kind of mathematics pertaining to theoretical physics according to a pluralist attitude (Mach 1896, chapter VIII).

but an independent theory very different from Newton's mechanics. However, owing to the long period of clarification of its higher mathematics – its differential equations have been solved around the mid-century –, it was characterized as an immature theory.

The problem vanished when (in 1865) Optics was included as a particular case in the new theory, electromagnetism. In its early stage of the historical development, this new theory presented a conflict with the accredited paradigm. Several phenomena and theoretical notions of both electricity and magnetism were divergent from Newton's ones. In particular, Faraday's basic notions (in particular, the concept of field of forces) were odd with respect to the Newton's notions. Yet, has included all these novelties inside a theory that assumed two vector fields (electric and magnetic) as the basic notions by means of which all possible laws are derived from four differential equations calculated in the infinitesimal neighborough of each point of the space. Hence, this theory reiterated Newton's kind of mathematics, relying on actual infinity as well as Newton's theory deductive organization. This new theory confirmed the Newtonian paradigm, although according to an enlarged theoretical version (e.g. four differential equations on four notions instead of one equation only on three notions), or maybe in view of a further synthesis of a Newtonian kind, according to Maxwell persistent hope.

Yet, in the mid of the XIX century the birth of thermodynamics proved that a non-Newtonian theoretical physics was possible. But the elementary mathematics of thermodynamics was underestimated as an insufficient attempt to introduce a more sophisticated mathematics. As a consequence, thermodynamics was relegated to the status of a naive phenomenological theory.

Moreover, at the same time the kinetic theory of gases born through the conservation energy law. Its birth resulted to be postponed of one century owing to Newtonian prejudices, in particular that of the notion of a hard body, which precluded the conservation energy law, as well as the need of using continuous variables, and hence forces (Drago 2014a). However, this theory is a merely particular case of the subsequent theory, statistical mechanics, where theorists planned to re-establish mechanics as the basic theory of the entire physics. Hence, they made use of continuous variables mathematics for proving the time evolution of a discrete physical notion, i.e. the microscopic entropy (Boltzmann's H-theorem). As a consequence, the theorists have considered a new theory that was different from Newton's – e.g. electromagnetism – as a mere extension of the Newtonian paradigm.

4. Einstein's "revolution". The re-birth of Galilei's dichotomy of the two kinds of infinity

In 1905 the paper which started quantum theory (Einstein 1905a) and which was called by Einstein (1905b) "the most revolutionary paper", showed that two mutually incompatible attitudes in the theoretical physics of light phenomena are possible. i.e. the attitude of the discrete mathematics and that of continuous mathematics. For his theory Einstein chose – against Maxwell and Newton – the "discrete", i.e. potential infinity.

Twenty years later, the author of the first formulation of quantum mechanics, Heisenberg, relied his theory upon matrix algebra; hence, he reiterated the same choice for potential infinity, this time for an entire formulation of the theory. Notice that in the same year Schrödinger instead chose to base his formulation on a differential equation of mathematical physics, i.e. actual infinity.

Few years later these two first formulations of quantum mechanics were generalized by the continuous Dirac-von Neumann theory. Since subsequently it resisted to all attempts (see e.g. Einstein's) to suggest an alternative formulation to it, a unity was again established and hence any choice was excluded.[2]

On another hand, the ambiguous interpretation of the electromagnetic induction led to a contradiction between Newton's mechanics and Maxwell's electromagnetism. Einstein's genius was to "conciliate" this conflict by suggesting a new theory, special relativity, at the cost of transforming Newton's mechanics, which was subjected to Lorentz' group. Hence, the comparison of two scientific theories concerning the same experimental phenomena acquired one more possibility: a conciliation through the invention of a new theory that generalizes one of the two contradictory theories.

However, quantum mechanics' laws are different and even in contradiction with classical laws (see e.g. Pauli's exclusion principle). The same occurs in special relativity; space-time is not the Euclidean space of Newton's mechanics. How put a remedy to these apparent contradictions? It was suggested that the historical development of theoretical physics occurs in a concentric way, i.e. subsequent theories include the previous ones concerning the same field of phenomena. As a proof, in quantum mechanics it was offered the existence of a limit process $h \rightarrow 0$ which is claimed to regain classical physics. But, first of all, one has to note that the mathematical process of a limit cannot include a change on the kind of infinity (as well as some other basic notions of the foundations of physics). In addition, it is well known that this limit gives only the Hamilton-Jacobi formulation, surely not Newton's formulation. Also in special relativity the limit $c \rightarrow \infty$ does not lead back to Newton's formulation, which relies on actions-at-a-distance (which imply the actual infinity in the interaction velocity), an hypothesis at all incompatible with special relativity (Goldstein 1980, p. 332). Rather this limit leads to e.g. L. Carnot's mechanics of contact interactions[3] (Scarpa 2002). These facts prove that *it is false that all formulations of classical mechanics are mathematically equivalent* (as believed even by Mach). We have to conclude that the above two limits, rather than giving theoretical power to both theories, show the insufficiency of the dominant philosophical

[2] Yet, Hanson (1961) contested the philosophical correctness of the operation of unification.

[3] Moreover, Bunge (1954) showed that, among all mechanics' formulations, Lagrange's one plays a special role because it is a general scheme, which is applicable to the entire theoretical physics. Hence, its range of validity is incomparably wider than Newton's.

conception, i.e. the concentric view of the historical development of theoretical physics.[4]

A further divergence has to be remarked; the two new theories, special relativity and quantum mechanics, are mutually incompatible, although each of them claims to include as particular cases the same classical theories. If one wants to interpret, as most scholars do, the differences between these theories as conciliable, i.e. as theoretical phenomena of *scientia condenda*, he has to believe that the question will be solved in next years. Yet, being elapsed ninety years from the births of these theories, I consider this belief as a naive hope rather than a realistic forecast.

5. The parallel birth of choosing the two kinds of infinity in the history of mathematics

In the history of mathematics parallel events occurred. Leibniz invented the infinitesimal analysis by founding it on the actual infinity; indeed, in Leibniz' opinion the symbol of the infinitesimal summarizes an actual infinite number of ideas (Brunschvigc 1923, part I, Livre III). However, he later aimed at obtaining, yet unsuccessfully, the same results through a calculus of finite quantities (Robinson 1960, chapter X).

Before XIX century all mathematicians rejected an explicit use of the actual infinity, apart from the indispensable infinitesimals. However, after Gauss the birth of a great number of abstract theories led mathematicians to make use of the actual infinity. Against this trend only Kronecker reacted; he aimed at founding the mathematical research only on integer numbers. Most mathematicians considered his program as suggested by a backwards attitude.

Yet, after few decades, in 1905 an obscure mathematician, motivated by mystical considerations, L.E.J. Brouwer, wrote a revolutionary program aimed at re-founding the entire body of Mathematics (and also Logic) on constructive techniques of the potential infinity (Brouwer 1975). According to him only the constructive part of mathematics had to survive. His offering new constructive versions of several previous results, together with his rejecting some important results as undecidable by constructive means, impressed some mathematicians. Even Hermann Weyl, although Hilbert's assistant, was persuaded that Brouwer's program was doomed to prevail. On the other hand, Hilbert had launched a program for founding mathematics (and the entire science too) on idealistic mathematical notions ("they are like the fists for a boxer"), among which the actual infinity. Since he considered Brouwer's program wrong, his reaction was vehement. Since each of them suggested an exclusive (win-lose) choice, a harsh intellectual battle has followed. At present time, a pacification without a clear appraisal on the past debate arrived (Martin-Loef 2007). However, Brouwer's program for a new

[4] A "spontaneous" discovery of this dichotomy at the level of mathematical techniques employed by the physical theories is (Barut 1986). This paper underlines the two different roles played inside a physical theory by the differential equations (called by him "dynamics") and the symmetries. They correspond *grosso modo* to the actual infinity and the potential infinity.

mathematics was formally accomplished by two independent scholars (Markov 1962, Bishop 1967). As a consequence, at present time no mathematician can exclude constructive mathematics from the valid theories, although it obtains different results from classical ones (e.g. undecidabilities).

The previous question (choice or conciliation?) may be suggested also for the divergences among the mathematical theories. Yet, since a formal conciliation of potential infinity with actual infinity is an apparently impossible task, no forecast of a future mathematical theory, which includes both kinds of infinity, is possible. Hence, *one has to accept the present situation of two irreconcilable foundations of mathematics; hence, a choice on them about the kind of infinity is unavoidable.* As a consequence, a pluralism was born about not only the various formulations of some single theory, but also about each possible theory relying on mathematics.

6. One more dichotomy: the kind of the organization of a theory

A similar story is that of the choice on the two kinds of organization of a scientific theory.

It is well known that in 320 BC Aristotle presented the model of a deductive science (Beth 1959). Soon after, Euclid organized his geometrical theory according to this model. Then, along two millennia this theory was an exemplar of the correct organization of a scientific theory; also because in 1687 Newton reiterated this model in the most important theory of classical physics, i.e. mechanics. This kind of organization was the other essential element – beyond the actual infinity – of the Newtonian paradigm in theoretical physics.

Yet seventy years later, in the *Encyclopédie Française* D'Alembert (1770-1775, volume V, p. 504) suggested that a deductive ("rational") theory presents in all cases "some holes"; in his opinion an "empirical theory" is more suitable for scientific theories. Few decades later, L. Carnot devoted two pages (L. Carnot 1783, pp. 101-103) to illustrate the two alternatives of this choice. He chose to found all his theories (geometry, calculus and mechanics) according to the "empirical" model, although he admitted as valid also the alternative model. Also in this case he suggested a pluralist attitude. Also the founder of non-Euclidean geometry, Lobachevsky, shared this pluralist attitude; he organized in an "empirical" way (or better, by posing problems to be solved, rather than axioms) each of his five works on this subject.

Unfortunately, most XIX century scientists have depreciated the "empirical" organization as a too informal way to arrange the elements of a theory. Later, when Hilbert suggested to axiomatize all scientific theories, almost all mathematicians believed that the deductive model of a theory was the only possible one.

Yet, in the history of theoretical physics, mathematical physics – according to which all physical laws are deduced from differential equations according to the Aristotelian organization – has been rejected by both the theory of quanta and special relativity, not only because they do not have been derived from differential equations, but also because they originated as "empirical" theories (i.e. without axioms).

Furthermore, in the XX century history of the foundations of mathematics, the birth of intuitionism on one side, and Goedel's theorems on another one, suggested that an axiomatic theory cannot grasp the entire content of a mathematical theory, even plain Arithmetic. Hence, what failed was the common belief that the Aristotelian model of organization is unique. Yet, no mathematician recalled D'Alembert-Carnot's distinction between the two different ways to organize a theory. Rather, three theoretical physicists, i.e. Lorentz, Poincaré and Einstein, re-discovered by ingenuity an alternative organization of a physical theory (Flores 2004). However, each of them suggested a little number of characteristic features of the new model of organization.

Some years ago an alternative model of organizing a scientific theory was recognized. One of us has extracted the features of this model by comparing all the scientific theories that have been presented by their authors in a different way from the Aristotelian model. An ideal model of the alternative kind of organization was obtained (Drago 2012). Thus, a new dichotomy in the foundations of science has to be added to the previous one.

Moreover, it was discovered that this dichotomy corresponds to a dichotomy in the foundations of mathematical logic; indeed, while the Aristotelian organization is governed by classical logic, the alterative organization is governed by an alternative logic, the intuitionist one. This logic started by a Brouwer's paper in 1908 and around twenty years later was formalized. It progressively gained relevance, so much that since the 1960s it was considered on par with classical logic; hence, a pluralism of the kinds of logic was established. In the meantime, several more kinds of logic (modal, minimal, non-monotonic, paraconsistent, fuzzy, etc.) have been formalized. At present time no one logics of this variety can be excluded as irrelevant (Gabbey, Kanamori, Woods 2012).

In conclusion, also this choice on the kind of organization, or equivalently on the kind of logic, is unavoidable. Remarkably, already Lorentz stressed that one has to take "a choice" on the kind of organization (Lorentz 1900, p. 33). In addition, since this dichotomy pertains to the foundations of logic, a choice on this subject concerns whatsoever scientific theory.

7. Philosophical pluralism of scientific theories

As a general conclusion, in the foundations of science two formal dichotomies – concerning the infinity and the organization of a theory or, equivalently, logics – were born. *We have to conclude that science includes some choices, which are not only choices of philosophical nature, but also of formal nature, concerning the foundations of science*.[5]

Yet, some philosophical prejudices obstruct a full recognition of these choices. Although the distinction in the various radically divergent kinds of logic is recognized as unavoidable, however by denying any relationship between logic and the real world,

[5] Kuhn (1969) overlooked the possibility of a choice two times. He represented a paradigm shift through a unexplained *Gestalt* phenomenon. Subsequently he avoided representing the choice – performed in a first time by Einstein and then by the scientific community – for the discrete mathematics in theoretical physics.

all relevant implications of these choices are avoided. Actually this attitude is justified by only a Platonist attitude on the entire logic. Instead, as a fact, Computer Science has abandoned classical logic for applying several kinds of non-classical logic. In addition, it is well known that Quantum Mechanics rejects classical logic.

Since the mid of XX century the notion of potential infinity, which previously has been considered as a merely philosophical notion, has based a formally well-defined mathematics (Markov 1962, Bishop 1967). The consequent sharp divide in the foundations of mathematics led eventually the mathematicians to recognize two "schools" on the philosophy of the mathematics. However most mathematicians have assumed an "ecumenical attitude" (Meschkowski 1965, chapter 10, footnote 1). According to these mathematicians the dichotomy on the kind of mathematics is a mere difference between two abstract cases, as the dichotomies Truth/False or Good/Evil in the Olympus of the Ideas. Hence, they feel themselves free to work in each of the two kinds of mathematics, classical and constructive, as an extension of the variety of the numerous mathematical theories. This attitude is a typical Platonist one of who considers mathematics a purely formal construct.

However, the above dichotomies have been ignored by the philosophers of physics, because along centuries the theoretical physics was dominated by a paradigm, according to which a scientist has to follow technical rules only. It is remarkable that – as we saw in the sections 4 and 5 – a choice between different theoretical formulations emerged each time the scientists have discovered theories outside the Newtonian paradigm. A choice eventually decisively emerged when both mathematics and logics allowed innovations – i.e. inner dichotomies of formal nature – which were excluded by the Newtonian paradigm. Some years ago it was proved, through the formal constructive mathematics, that there exists a mathematical divide among the set of constructive formulations of a physical theory and the set of its non-constructive formulations (Drago 1986, Da Costa, Doria 1999).[6] Hence, in theoretical physics there exists a "dichotomy" about the kind of mathematics – as about the case of the light Einstein wrote in the 1905 paper on quanta.[7]

As a matter of fact, already at the birth of Newton's paradigm a great mathematician, physicist and philosopher, Leibniz, recognized two labyrinths in our mind; the labyrinth of either potential or actual infinity; and the labyrinth of either freedom or law (Leibniz 1989). The latter labyrinth translates the dichotomy on the kind of organization in subjective terms. Indeed, the former alternative ("freedom") allows a free search for discovering a new "empirical" method for solving the problem stated by the theory, while the latter alternative ("law") obliges to obey a list of laws, as it occurs inside the development of an Aristotelian organization. In such a way one obtains exactly the two previous dichotomies. After three centuries Leibniz' suggestion

[6] These results are presently exorcised by claiming the "indispensability" of classical mathematics in the applications to reality. Yet, both computer science and theoretical biology – born without calculus – make use of discrete mathematics for stating their basic results.

[7] Since a long time in philosophy of science there exists a debate on the unity of science. The neo-positivists claimed a total unity; owing to this belief they had planned an *Encyclopedia of the Unified Science* (which however was unsuccessful). Several philosophers have undermined this thesis (e.g. Agassi 1969). Yet, most of the latter ones ignore the different formulations of a scientific theory.

of the two labyrinths was vindicated by the historical development of mathematics and logics.

8. General considerations on the ethics of the two dichotomies of science

Leibniz correctly qualified them as labyrinths, because the reason alone cannot solve the problem of how to choose on them; hence, they are dichotomies on which a non-scientific motivation only can lead to decide.[8]

Notice that these choices first of all pertain to a single scientist, who is building an entire new theory; e.g. to Newton when he was building his mechanics' theory; to Einstein when he was building his special relativity. It is unavoidable to conclude that these choices, being free decisions taken by a scientist, have ethical meaning for the scientist himself. In some cases the alternative choice (expressed through the foundation of an alternative formulation) was suggested even a century later (e.g. L. Carnot's formulation which is alternative to previous Newton's formulation). Yet, this time lag does not influence the ethical nature of the opposite choices taken by both scientists. Hence, *theoretical physics includes as its constitutive part also ethical decisions taken by the single founders of theories.* In addition, owing to the antecedent decisions taken by the founders of mutually alternative formulations of a theory implies that at present whatsoever scientist choosing to work inside a specific formulation takes – although implicitly – the corresponding ethical choices.

Surely, a dilemma is a very elementary subject of an ethical system. Yet, if the dilemmas-dichotomies are two, and they are mutually independent, then a primordial, but effective ethical system is obtained. This system is relevant for two reasons. First, the four couples of choices on the two dichotomies compose a compass; each couple addresses the mind to follow a specific direction inside the sea of innumerable scientific theories. Even more importantly, provided that one suitably adapt the philosophical meanings of the choices, similar dichotomies to those in science hold true in the foundations of ethics (Drago 2000). Hence, one may stress that although scientist's ethics is a minimal one, this ethics system echoes the foundations of the entire ethics system.

[8] This point was equivocated by Kant who thought to have proved the basic tenets of each of the two alternatives and hence to have obtained an antinomic contradiction between them. This misinterpretation led him to make recourse to a formal viewpoint, which moreover was based upon metaphysical pre-conceptions (e.g. the perception of space through a "pink eye-glasses"). The discovery of non-Euclidean geometries denied this philosophy of knowledge (Drago 2014b).

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