Galileo’s Free Fall into History of Physics & Nature of Science Teaching

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Abstract: In *Discorsi e Dimostrazioni Matematiche su Due Nuove Scienze,* Galileo Galilei (1564-1642) described in detail his experiments about free fall motion, the motion along an inclined plane and the parabolic motion. By notes on the subject Stillman Drake (1910-1993) suggested its historical account. Recent works proposed that the notion of time received a mathematical treatment and a physical measurement entering into the equations of the fall of the bodies. Our paper aims to describe an historical and *Nature of Science*/experimental path on Galilean physics and mathematics. One of us (VC) is currently working at a high school in Naples (Italy) with the following educational aspects of the present research: analysing Galilean sources (about free fall) and secondary literature on the subject. The main idea is starting from the analysis of the pendulum motion and of the water clock essential to perform the measurement of time – right in the historical context.

Keywords: Galileo, Free fall, Nature of Science Teaching, History of Physics.

1. Introduction

The overall aim of this work is to address the Nature of Science (NoS)-teaching of science (physics, physics-mathematics) and educational aspects of the teaching-learning of physics in high school and University, studying both didactic and pedagogical questions within/by means of history of science/Nos (physics, mathematics): priority to situate the learning difficulties experienced by pupils at the core of educational activity; understanding how historical foundations of science can be used for teaching or pedagogical purposes; proposing to the students an appropriate educational path in a physics measurement workshop while respecting the didactics of science (physics-mathematical modelling) by means of a historical interdisciplinary approach to the disciplines of science education.

2. Galilean sources: *De Motu* and the Ms. 71 of Galileo's Manuscripts (Gal71)

De Motu Antiquiora is the title of six Latin documents concerning the movement of bodies subject to gravity (considered only as a quality) and some of its applications including the fall in a medium, along an inclined plane and his first investigations into the motion of projectiles (still linked to medieval tradition). These writings are contained almost entirely in the Gal71 manuscript conserved at the *Biblioteca Nazionale Centrale di Firenze* (BNCF). They were approximately written by Galileo in Pisa between 1589 and 1592 and were published for the first time only in the second half of the 1800s. Favaro collected them together in his first volume of the National Edition of the Works of Galileo (Favaro 1890-1909, I, pp. 251-419). They consist of a treaty of 23 chapters (pp. 251-340), a reworking of the first two chaps (pp. 341-343), an essay in ten chapters (pp. 344-366), a dialogue (incomplete) concerning the themes of the treaty (pp. 367-408), some notes not included in Gal71 manuscript (pp. 409-417) and the work plan (pp. 418-419). At that time, the motion could be supposed to be natural or violent, according to the Aristotelian school. The natural motion consists of bodies spontaneously moving towards a natural place; the violent motion is determined by a force (Ross 1930, *De Caelo*, 276a). Galileo – after criticizing Aristotelian approach – tried this conceptualisation, arguing Archimedes' hydrodynamics (Chalmers 2017). Galilean approach consisted of a speed of a moving body – upwards or downward – by natural motion (uniform) that is proportional to the difference between the specific weight of the body and that of the surrounding medium (Favaro 1890-1909, I, pp. 271-273);[1] rejecting the idea of Aristotle who believed that the falling speed is proportional to the weight of the body (Ross 1930, *Physica*, 216a 13-16); inversely proportional to the density of the medium (Ross 1930, *Physica*, 215a 31- 215b 12) when comparing with *Discorsi* (Favaro 1890-1909, VIII, pp. 105-117). Galileo, by means of his Archimedean approach, considered as possible the motion of a body in vacuum, with a maximum but–not–infinite speed as Aristotle's theory had predicted. In *Discorsi*, Galileo renounced to any explanation of a dynamic type but further developed his considerations on the fall of bodies of the same form but of different nature in fluids of different resistance, arriving at a correct description of the motion (as accelerated in a first phase and then at constant speed in the second one) arguing by decreasing the resistance of the medium – accordingly with the vacuum, the fall times do not depend on the mass of the bodies (Favaro 1890-1909, VIII, pp. 117-119). In NoS, this is a very interesting example of preliminary analysis of a phenomenon and quantities. It helps to understand what are the fundamental procedures followed by scientists in the formulation of a physical law.[2] The criticism shared by Galileo of the Aristotelian theory concerning the active role of the medium, to justify violent motion (Ross 1930, *Physica*, 215a 16-18) must also be attributed to Benedetti. Both scientists resorted to the notion of *vis impressa*, very close to the medieval one of *impetus* (and to the modern one of *momentum*) that did not require the application of any additional force to keep a body in motion. The Archimedean approach also helped to determine the right expression of the upward force necessary to balance the effect of gravity; see Fig. 1 (left).

[1] Hardly was the calculation between two non–homogenous quantities like space and time. In fact, in the period of time, they were commented as non–possible to do.

[2] The Archimedean approach and the conclusion that bodies of different masses but of the same type fall with the same speed can be found in the years immediately preceding even in the work of Giambattista Benedetti (1530-1590); see (Benedetti 1953).

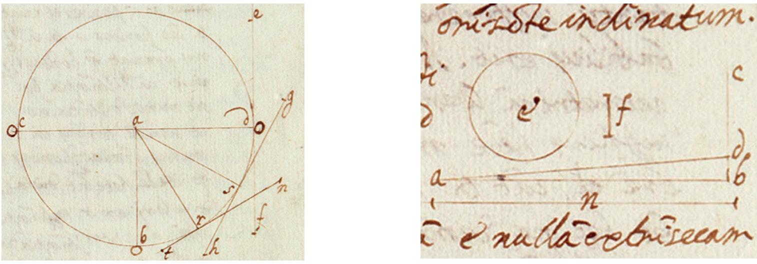

Fig. 1. Left: proofing of the upward force necessary to balance the effect of gravity along an inclined plane. Manuscript Gal 71. Detail of folio 95r. Right: by decreasing the slope of the plane *ad* until the horizon, the sphere *e*, if subject only to its weight *f* and to no resistance, can be moved by a force less than any given force. Manuscript Gal 71. Detail of folio 97r (with kind permission of *Biblioteca Nazionale Centrale di Firenze*).

In fact, it has to be observed that "the same weight can be drawn up an inclined plane with less force than vertically, in proportion as the vertical ascent is smaller than the oblique" (Favaro 1890-1909, I, p. 298; Galilei 1960, p. 65).[3] Galileo considers bent levers moving along a circumference and assumes the motion of a body in a point of the lower quadrant of the circle to be equivalent to that of a body descending on a sloping plane tangent to the circle in that point.

A consequence of the theorem is an earliest formulation of the inertia principle. Indeed, a spherical body "that does not resist motion […] on a plane parallel to the horizon will be moved by the very smallest force, indeed, by a force less than any given force" (Favaro 1890-1909, I, pp. 298-299; Galilei 1960 pp. 65-66); see Fig. 1 (right). Therefore, while the spontaneous downward motion along an inclined plane must be considered to be natural and the upward movement to be forced, the motion along a horizontal plane is neither one nor the other but what Galileo called "neutral motion" (Galileo 1960, p. 67, ft. 9). Galileo gave another example of "neutral motion" as that of a body that moves undisturbed with uniform circular motion (circular inertia[4]) around the centre of the Universe as the skies and the stars do – in a still geocentric vision – along a concentric sphere with the Earth without moving away or approaching its centre (Favaro 1890-1909, I, p 305). In Galileo's opinion, a uniform rectilinear motion, on the other hand, occurring in a plane tangent to this sphere, had the problem of not being

[3] Galileo gives another proof of the same theorem in *Le Meccaniche* (Favaro 1890-1909, II, pp. 181-183). This paper, devoted to the science of weights (i.e. equilibrium), is also relevant for the study of motion. See for example the introduction of the concept of *momento* which has both static (Favaro 1890-1909, II, pp. 17-28, 159) and dynamic (Favaro 1890-1909, II, p.68) connotations (Pisano, Gatto 2021, pre-print).

[4] Galileo returned to the concept of circular inertia in *Le Mecaniche* (Favaro 1890-1909, II, pp 179-180), more explicitly in *Seconda Lettera sulle Macchie Solari* of 1612 (Favaro 1890-1909, V, pp. 134-135) and in *Dialogo sui Due Massimi Sistemi del Mondo* (Favaro 1890-1909, VII, pp. 56-57). In his maturity, in *Discorsi* we find that the terms of principle of inertia are very similar to the modern ones: "[…] any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed, a condition which is found only on horizontal planes" (Favaro 1890-1909, VIII, p. 243; Galilei 1914, p. 215). However, lacking the theory of gravitation, the Pisan scientist would never be able to solve the problem of uniform circular motion (Favaro 1890-1909, VIII, pp. 283-284; Galilei 1914, pp.261-262).

able to preserve for a long time its steady speed because its points were not equidistant from the centre of the Earth (Favaro 1890-1909, I, p. 301).

3. Time and its measure. The Law of Quadratic Proportionality

A great innovation at the origins of the modern science of mechanics is the emphasis given by Galileo to the notion of time achieved through two complementary approaches – mathematical and measuring (Abattouy 1992, p. 119; on Galileo see also Palmerino 2018).[5] In order to prove that the naturally descending bodies follow a uniformly accelerated motion, Galileo claims to have made a bronze ball for a clean and smooth canal, measuring time by a water clock (Favaro 1890-1909, VIII, pp. 212-213). The constant-flow chronograph, coupled with a fairly sensitive weight scale, was a particularly useful tool for Galileo because it can measure short intervals of time in a continuous manner in which the uncertainty of the measure is essentially due to the investigator's reflexes (Vergara Caffarelli 2009, pp. 211-212). In the letter to Giovan Battista Baliani dated 1 August 1639, Galileo emphasises the importance of this measuring instrument, but also that of using the pendulum for time measurements (Favaro 1890-1909, XVIII, p. 76-77). Galileo announced his discovery of the pendulum isochronism in the letter to Guidobaldo del Monte dated 29 November 1602 (Favaro 1890-1909, X, pp. 97-100) and described, first in the *Dialogo* (Favaro 1890-1909, VII, pp. 475-477) but, above all, in the *Discorsi* (Favaro 1890-1909, VIII, pp. 128-130, 139-140), some simple experiments about the pendulum that have great educational value.[6] So we can suppose that Galileo would use the pendulum to get time intervals all equal to each other in order to experimentally verify the so-called law of odd numbers,[7] stating that a naturally descending body starting from rest, during equal intervals of time, traverses distances which are related to each other as the odd numbers beginning with unity (1, 3, 5, …), that is, measuring the spaces all from the quiet position and the corresponding time intervals, the spaces traversed are in the ratio of the squares of the times.

[5] We find the concept of time in the description of motion only in the formulation of Kepler's third law and in the mathematical description of the spiral as a function of time by Archimedes (Abattouy 1992, p 130; Pisano, Bussotti 2017; Pisano, Agassi, Drozdova 2017) as the composition of a uniform rectilinear motion and a uniform circular one (Heath [1987] 2002), p. 154).

[6] Pendulum properties can be studied as was done by Galileo through three simple experiments: two pendulums – of different masses, with different amplitudes or different lengths – are made to oscillate together. The oscillation time changes only in the last case (Galileo suggests a length of four times that of the other one, obtaining an oscillation time that is twice of the other).

[7] Drake (1975. pp 100-101) suggested that on folio 107v calculations and diagrams are linked to experimental data, in order to confirm the law of odd numbers: "the figures listed in the third column at the top left represent very nearly the distances from rest of a ball rolling down an inclined plane at the ends of eight equal times. The distances were actual, not theoretical, since they differ slightly from the products of the first number, 33, by the square numbers 1, 4, 9, …, 64". See Fig. 2 (left). Doubts about this interpretation were expressed in (Renn *et al.* 2000) and (Damerow *et al.* 2004, p. 167, ft. 83).

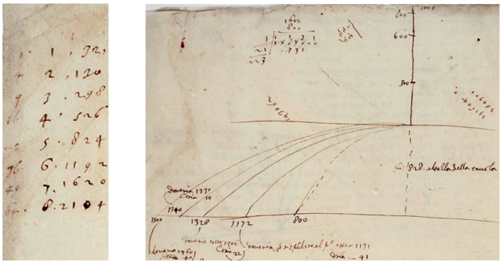

Fig. 2. Left: data about the odd numbers law. Gal72. Detail of folio 107v. Right: the measures of the ranges of the projectile motion with horizontal initial velocity. Manuscript Gal 72. Detail of folio 116v (with kind permission of *Biblioteca Nazionale Centrale di Firenze*).

4. The Galilean Manuscript 72

The Galilean Ms. 72 is conserved at the *Biblioteca Nazionale Centrale di Firenze*. It includes his handwritten notes on motion, almost all in the original hand of Galileo (1602-1637, ff. 33r-194r) and some other sheets concerning with other topics such as an incomplete copy of *Le Mecaniche* (Galilei 1602-1637, ff. 9-26). They range from 1602 to the publication of the *Discorsi*. Favaro published most documents in the footnotes to *Discorsi* or in the Fragments relating to the manuscript (Favaro 1890-1909, VIII, pp. 363-448). Drake published them entirely in a probable order in 1979 (Drake 1979). Among the most significant folios for educational purposes we can certainly include the previously quoted folio 107v relating to the study of naturally accelerated motion and the folio 152r where the conceptual change is attested by a falling speed proportional to space to a speed proportional to time (Damerow *et al*. 2004, 180-197). His experimental notes on the parabolic motion of the projectiles are of particular interest. It is already in Tartaglia's view that the launch angle for the maximum range of the projectile is 45° (Pisano, Capecchi 2015, p. 39; Pisano 2020). The first studies on the parabolic trajectory are due to Galileo and Guidobaldo del Monte (Renn *et al.* 2000). Accordingly to Drake and MacLachlan (1975), the most significant Galileo's experiment can be considered an indirect test of the principle of inertia. The top of folio 116v (see Fig. 2, right) represents the trajectories of a ball that, after falling from an inclined plane, is deflected to move initially in the horizontal direction. The horizontal motion, ignoring the air resistance, obeys the Galilean inertia principle that, in the absence of forces, the ball must continue to move with a straight-line motion at constant speed. The ball in the vertical direction is subject to the weight force. The result of this composition of movements is a parabolic motion. Along a vertical line Galileo recorded the numbers 300, 600, 800, and 1000. They are the heights from which a ball is descending along an inclined plane (that has not been drawn). The unit of measure taken by Galileo is the point, which is equivalent to 0.9 mm. The table is placed at a height of 828 points. Along the horizontal axis are reported the distances, in points from the vertical, to which the ball touches the ground for the different heights of the inclined plane. The expected values of such distances (with the deviations from the experimental values) cal-

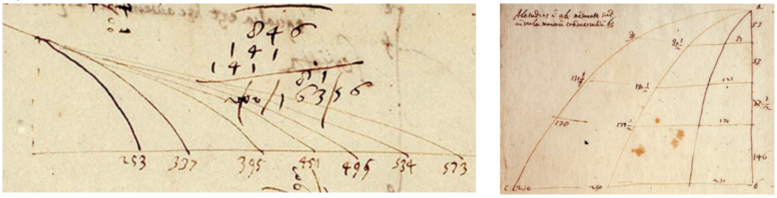

Fig. 3. Left: measured ranges of a falling motion with initial velocity in oblique direction. Manuscript Gal 72. Detail of folio 114v. Right: reconstruction of the trajectories by a horizontally launched ball. Manuscript Gal 72. Detail of folio 81r (with kind permission of *Biblioteca Nazionale Centrale di Firenze*).

culated by Galileo are also indicated, assuming the conservation of velocity in the horizontal direction, proportional to the square root of the height of the inclined plane.

In order to visualize the whole trajectories (not just the impact point with the ground), Galileo recorded the intersections of these with multiple parallel planes placed at different heights. As shown in folio 81r (see Fig. 3, right), he examined the behaviour of a ball that, after falling down along the inclined plane, describes a curved trajectory until it strikes with a plane surface. Galileo recorded the impact points with the utmost precision possible for different planes, in order to obtain a set of points in the space whose interpolation assumes the shape of a curve, corresponding geometrically to a parabola and nowadays described analytically by a second-degree equation (Hill 1988). On this issue, the analysis of folio 117r is also very attractive (Naylor 1976; Hill 1979).

5. A NoS approach

Other researchers performed, with different purposes, the experiments based on Galileo's writings and laboratory notes (Settle 1961; Vergara Caffarelli 2009; Cerreta 2014). At the *Liceo Scientifico "Sbordone"* in Naples a comprehensive experimental educational path was identified by one of us (VC) as synthetized in Table 1.

Acknowledgments

We express our sincere thankfulness and appreciation to Madame Susanna Pelle, who, on behalf of *Settore Manoscritti - Servizio Riproduzioni e Diritti* of *Biblioteca Nazionale Centrale di Firenze*, gave us the permission to use images of Ms Gal. 71 and Ms. Gal. 72. We thank the Staff, particularly Mme Maria Rosaria Bellavita, of the *Biblioteca* "*Roberto Stroffolini*", Department of Physics "*Ettore Pancini*" at the University of Naples "Federico II". Finally, we are grateful to *Liceo Scientifico Sbordone*'s Principal, Maria Antonella Caggiano, for supporting our project and allowing its further experimentation.

Misconceptions of the students	a.	Dependence of pendulum swing time on mass and amplitude.
	b.	Dependence on the mass of the time of fall of a body in the void.
	c.	Direct proportionality between the oscillation period of the pendulum and the length of the string.
	d.	Direct proportionality between the speed in a free fall motion and the distance travelled.
	e.	The force must continue to act to keep a body in motion.
	f.	Direct proportionality between the space travelled along an inclined plane and the time interval spent.
	g.	Trajectory of the motion of the projectile consisting of a straight line until the initial thrust is exhausted and an almost vertical line due to the weight effect.
	h.	Straight trajectory of the projectile falling motion with oblique downward velocity.
Problem-posing	a.	How could we measure time if we were Galileo?
	b.	Does the free fall of bodies in a vacuum depend on the mass?
	c.	In the free fall motion, how does the distance travelled depend on the time?
	d.	How is it possible to obtain a motion at constant velocity when a body is not subject to forces?
	e.	What are the trajectories of a projectile?
Problem-solving: the units of the learning path	a.	Let us swing simultaneously two pendulums of different masses, with different amplitude or with wires of different lengths. It is observed that the period of one oscillation depends only on the length of the wire. The fact that it does not depend on the amplitude of the pendulum oscillation during the motion allows us to use the period of oscillation as unit of time measurement (a pendulum length of 25 cm is chosen). To obtain a continuous measurement of time we build a water clock.
	b.	We drop small balls of different masses first in oil, then in water, then in air by extrapolating their behaviour in vacuum. As a further study suggested by Galileo's writings, we can verify if the terminal velocities of objects as they fall through a fluid are proportional to the differences between the specific weight of the body and that of the surrounding medium.
	c.	We slow the free fall movement by using a 6.75 meters long inclined plane so that we can study motion. We do two experiments: we fix the times that are marked by the pendulum and measure the spaces travelled; we fix the spaces that must be travelled and measure the corresponding time intervals using the water clock.
	d.	We drop a little ball on an inclined plane, varying the height and thus its final speed; then the fall motion continues in air preserving a uniform movement in the horizontal direction. If the initial velocity of the ball in air is horizontally directed, we can verify the direct proportional link between the range and the square root of the launch pad height. We check whether it is possible to extend this study to the case of a initial velocity directed in an oblique direction.
	e.	We can measure the range of the ball at different heights of the impact point so as to reconstruct the trajectory that it is a parabola.

Table 1. The historical experimental learning path.

References

Abattouy M. (1992). *Deux aspects de la notion du temps dans l'École Galiléenne: Mathématisation et mesure*, in Bucciantini M., Torrini M. (eds.), *Geometria e atomismo nella scuola galileiana*. Firenze: Olschki, pp. 119-145.

Benedetti G.B. (1553). *Resolutio omnium Euclidis problematum aliorumque ad hoc necessario inventorum*. Venice: Bartholomaeum Caesanum.

Cerreta P. (2014). "La radice quadrata, un algoritmo nascosto nella gravità". *Giornale di Fisica*, 55 (1), pp. 45-60.

Chalmers A.F. (2017). *One hundred years of pressure*. New York: Springer.

Damerow P., Freudenthal G., McLaughlin P., Renn J. (2004). *Exploring the limits of preclassical mechanics*. Second edition. New York: Springer.

Drake S. (1979). *Galileo's Notes on motion*. Firenze: Istituto e Museo di Storia della scienza.
Drake S., MacLachlan J. (1975). "Galileo's discovery of the parabolic trajectory". *Scientific American*, 232 (3), pp. 102-111.
Favaro A. (ed.) (1890-1909). *Le opere di Galileo Galilei: Edizione nazionale sotto gli auspici di sua maestà il re d'Italia*. Firenze: Barbera.
Galilei G. (1602-1637). *Opere di Galileo Galilei. Meccanica. Parte 5. Tomo 2. Manoscritto Gal. 72. Fondo galileiano: Biblioteca Nazionale Centrale di Firenze*. URL: http://www.bncf.firenze.sbn.it [access date: 30/06/2019].
Galilei G. (1914). *Dialogues concerning two new sciences*. Translated by Crew H, de Salvio A. With an Introduction by Favaro A. New York: Macmillan.
Galilei G. (1960). *On Motion and On Mechanics*. Edited by I.E. Drabkin and S. Drake. Madison: The University of Wisconsin Press.
Heath T. (ed.) ([1987] 2002). *The Works of Archimedes*. New York: Dover.
Hill D.K. (1979). "A Note on a Galilean worksheet". *Isis*, 70, pp. 269-271.
Hill D.K. (1988). "Dissecting trajectories: Galileo's early experiments on projectile motion and the Law of Fall". *Isis*, 79, pp. 646-668.
Naylor R.H. (1976). "Galileo: The search for the parabolic trajectory". *Annals of Science*, 33, pp. 153-172.
Palmerino C.R. (2018). "Discussing what would happen: The role of thought experiments in Galileo's Dialogues". *Philosophy of Science*, 85 (5), pp. 906-918.
Pisano R. (2017). *On the conceptualization of force in Johannes Kepler's corpus: an interplay between physics/mathematics and metaphysics*, in Pisano R., Agassi J., Drozdova D. (eds.). *Hypothesis and perspective in history and philosophy of science. Homage to Alexandre Koyré 1892-1964*. Dordrecht: Springer.
Pisano R. (2020). "A Tale of Tartaglia's Libro Sesto & Gionta in *Quesiti et Inventioni diverse* (1546-1554). Exploring the Historical and Cultural Foundations"'. *Foundations of Science*, 25 (2), pp. 477–505.
Pisano R., Gatto R. (2021). Galileo's *Le mecaniche*. English translation and critical commentaries. Dordrecht: Springer, preprint.
Pisano R., Bussotti P. (eds.) (2017). "Homage to Galileo Galilei 1564-2014. Reading iuvenilia Galilean works within history and historical epistemology of science". *Philosophia Scientiae*, 21 (1).
Pisano R., Capecchi D. (2015). *Tartaglia's Science of Weights and Mechanics in the Sixteenth Century*. Dordrecht: Springer.
Renn J., Damerow P., Rieger S. (2000). "Hunting the white elephant: When and how did Galileo discover the Law of Fall". *Science in Context*, 13, pp. 299-419.
Ross W.D. (ed.). (1930). *The Works of Aristotle*. Vol. II. *Physica*, *De Caelo*, *De Generatione et Corruptione*. Oxford: Clarendon Press.
Settle T.B. (1961). "An experiment in the history of science". *Science*, 133, pp. 19-23.
Vergara Caffarelli R. (2009). *Galileo Galilei and Motion*. Bologna: Società Italiana di Fisica.