Λ-Units and Λ-Mega Quantum of action in their historical context

Ludwik Kostro, ATENEUM – University of Higher Education, Gdańsk, prof. emeritus at Gdańsk University, Poland, fizlk@univ.gda.pl

Abstract: In Physics we are using conventional units, e.g. nowadays the SI system. Some physicists, among them G. J. Stoney, M. Planck, and Ch. Kittel have introduced into physics the so-called "Natural Units" determined by universal constants. Since several years I am trying to introduce a set of natural units determined by the three Einstein's constants c, $\kappa = \frac{8\pi G}{c^4}$ and Λ. Since long time in Relativistic Cosmology there are already used some units determined by c, κ and Λ: the Λ-force $= \frac{1}{\kappa}$; Λ-pressure of the physical vacuum $P_\Lambda = \frac{\Lambda}{\kappa}$; Λ-mass density $\rho_\Lambda = \frac{\Lambda}{\kappa c^2}$ and Λ- energy density $\rho_\Lambda\ c^2 = \frac{\Lambda}{\kappa}$. In my paper (Kostro 2018) there was published the whole list of Λ-units. I call them Λ- units because they are determined, for the first time, also by the cosmological constant. Among them there are the Mega Units (perhaps Mega Quanta): of Λ-Mass $M_\Lambda = \frac{1}{c^2 \kappa \Lambda^{½}}$, of Λ-Energy $E_\Lambda = \frac{1}{\kappa \Lambda^{½}}$; the Mega Quantum of Λ-Action $H_\Lambda = \frac{1}{c\kappa\Lambda}$. Since some of Λ-units can be interpreted as de Broglie like Λ-period $T_\Lambda = \frac{1}{c\Lambda^{½}}$, Λ-frequency $v_\Lambda = c\Lambda^{½}$, Λ-energy $E_\Lambda = H_\Lambda v_\Lambda$ and Λ-wave length $\lambda_\Lambda = \frac{H_\Lambda}{M_\Lambda c}$ and because between the all universal constants there ate correlations and interconnections we can ask the question whether in the Nature there are perhaps Λ-mega waves, Λ-mega fluctuations. We can ask also the question whether the unification of the Relativistic Cosmology with QM has to be done rather in the Large Scale then in the Small Scale. It will be also indicated that the crucial date in the evolution of our Universe i.e. its expansion acceleration about the 9 billion years after the Big Bang, coincides with the lambda time t_Λ ~ 9 billion years.

Keywords: universal constants, cosmological constant, natural units.

1. General data regarding the so-called "natural units" and their historical context

In Physics we use conventional units, e.g. nowadays the SI system. There were physicists (Stoney, Planck, Kittel and others) that introduced units called by them "natural units" because they are determined by universal constants which govern in our universe and are recognized as fundamental and important in the contemporary physics.

1.1. Some historical pieces of information about Stoney's Units

In 1874 the Irish physicist George Stoney (1826-1911), who is famous for his introduction of the term "electron" to describe the elementary unit of electricity and for his calculation of its value from Faraday's law of electrolysis, introduced his "physical units of nature" determined by c – velocity of light, G – Newton's gravitational constant and e - the elementary unit of electric charge. Stoney expressed them in the framework of cgs units. Here they are

$$l_S = \left(\frac{Ge^2}{c^4}\right)^{\frac{1}{2}};\ t_S = \left(\frac{Ge^2}{c^6}\right)^{\frac{1}{2}};\quad m_S = \left(\frac{e^2}{G}\right)^{\frac{1}{2}}$$

The scientific community has recognized his discovery of the "electron" the existence of which was proved experimentally in 1897 by Joseph J. Thomson (1856-1940). But Stoney personally was convinced that the discovery of his "natural units" is more important and therefore he published his paper with the title *On the Physical Units of Nature* (Stoney 1874, 1881 pp. 381-391).

In the framework of the SI system of units Stoney' s Units are given by

$$l_S = \left(\frac{Ke^2G}{c^4}\right)^{\frac{1}{2}} = 1.38 \cdot 10^{-36} m; \qquad t_S = \left(\frac{Ke^2G}{c^6}\right)^{\frac{1}{2}} = 4.605 \cdot 10^{-45} s;$$

$$m_S = \left(\frac{Ke^2}{G}\right)^{\frac{1}{2}} = 1.86 \cdot 10^{-9} kg$$

and the quantum of action $h_S = \frac{Ke^2}{c} = 7.704 \cdot 10^{-37}\, J{\cdot}s$

where $K = \frac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^{9}\, Nm^2/s^2$.

Many known physicist (e.g. Eddington, Schrödinger) indicated that the ratio of Stoney's quantum of action h_S and Planck's quantum of angular momentum $\hbar = \frac{h}{2\pi}$ gives the Sommerfeld fine structure constant $\frac{h_S}{\hbar} \sim \frac{1}{137}$.

1.2. Some historical pieces of information about Planck's Natural Units

At the turn of XIX and XX century, Max Planck (1858–1947) has not only introduced his very important constant $h = 6.626 \cdot 10^{-34}$ J s called by himself "the elementary quantum of action", but he has also, at the same time, introduced his Natural Units determined by three universal constants c – velocity of light, G – Newton's gravitational constant and his constant h (Planck 1899). The scientific community has soon recognized the importance of Planck's constant that became the quantization parameter of the new born Quantum Mechanics. But as regards Planck's Units the scientific community has longtime ignored them. So Planck during 12 years has added to all his papers his Units believing that the community will finally recognize also the importance of his Units.

Later in order to avoid in theoretical considerations the Mathematical Collapse of the whole Universe into a mathematical point the Planck's units were introduced *ad hoc* into modern cosmology.

Especially the cosmologists to avoid the mentioned collapse have recognized their importance and nowadays we speak even about the Planck's era existing at the beginning of the cosmic evolution.

That was done first using the Units introduced by Planck himself with

$$h = 6.626 \cdot 10^{-34}\, J \cdot s;$$

$$l_P = \left(\frac{hG}{c^3}\right)^{\frac{1}{2}} = 4.05 \cdot 10^{-35}\,\text{m}; \; t_P = \left(\frac{hG}{c^5}\right)^{\frac{1}{2}} = 1.35 \cdot 10^{-43}\,\text{s}$$

$$m_P = \left(\frac{hc}{G}\right)^{\frac{1}{2}} = 5.45 \cdot 10^{-8}\,\text{kg}; \quad \mathcal{T}_p = \left(\frac{hc^5}{k^2 G}\right)^{\frac{1}{2}} = 3{,}55 \cdot 10^{32}\; K^o$$

And later using Planck's Units with $\hbar = \frac{h}{2\pi}$. In such a way Planck's era became 2,5 times shorter and the governing initial temperature $\mathcal{T}_p$ became 2,5 times less high.

$$l_p = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = 1{,}6 \cdot 10^{-35} m\,; \quad t_p = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = 5{,}4 \cdot 10^{-44} s$$

$$m_p = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} = 2{,}17 \cdot 10^{-8} kg\,; \quad \mathcal{T}_p = \left(\frac{\hbar c^5}{k^2 G}\right)^{\frac{1}{2}} = 1{,}41 \cdot 10^{32} K^o.$$

1.3. Some historical pieces of information about Kittel's gravitational Natural Units for a concrete ponderable mass m_G

Ch. Kittel (1965, p. 302) has indicated (or better reminded) that for any given gravitational mass m_G its gravitational length, time and mass are given by

$$l_G = \frac{Gm_G}{c^2}\,; \quad t_G = \frac{Gm_G}{c^3}\,; \quad m_G = m\,.$$

In Kittel's set of units the quantum of action for concrete m_G is given by $h_G = \frac{G{m_G}^2}{c}$ and hence

$$m_G = \left(\frac{h_G c}{G}\right)^{\frac{1}{2}} \quad \text{and} \quad E_G = \left(\frac{h_G c^5}{G}\right)^{\frac{1}{2}}$$

where m_G is the gravitational charge of a particle, of a body, of a star or even of the gravitational matter embedded, according to the standard model, in a Hubble visible sphere. Note that the gravitational length l_G is in a strict relation with the so-called gravitational radius R_G called also Schwartzschild radius. In General Relativity we are dealing with three possibilities of R_G , indicated by S. Weinberg (1972, pp. 207-208). They depend on the used system of coordinates and metric.

$$R_{G(1)} = \frac{2Gm_G}{c^2}\,; \quad R_{G(2)} = \frac{Gm_G}{c^2}\,; \quad R_{G(3)} = \frac{Gm_G}{2c^2}$$

In the case $R_{G(2)} = l_G$ the metric is expressed in its harmonic form.

1.4. Some historical pieces of information about Natural Units determined by three Einstein's relativity theory constants c, κ and Λ

In the present paper I am going to introduce a toy-model of some domains of the physical reality in which the Einstein's cosmological constant Λ plays its important part. I call it toy-model because I shall play with universal constants recognized in our human physics. They are the players in the proposed game. My purpose is only to indicate some new aspects connected with the still hypothetical Dark Energy. Of course, the objective problems connected with Dark Energy cannot be resolved only by playing with constants. They must be resolved first of all by hard sophisticated mathematical work and by always more precise observations and experiments.

Nevertheless playing with universal constants we can sometimes arrive to interesting results. For example the coefficients $\frac{c^4}{G}$ and $\frac{c^5}{G}$ composed of two universal constants c and G used in General Relativity and Relativistic Cosmology have the dimensions of force and power and it is easy to show (Kostro 1999, pp.182-189; 2000, pp. 143-149) that these two coefficients have to be considered as the greatest possible limitary force F_{lim} and power P_{lim} in Nature

$$F_{lim} = \frac{c^4}{G} = 1.\,2107 \cdot 10^{44}\ N\,; \quad P_{lim} = \frac{c^5}{G} = 3.63 \cdot 10^{52}\ W\,.$$

Therefore e.g. we cannot construct an accelerator in which the increase of momentum per second and the liberation of energy per second could be greater then $\frac{c^4}{G}$ and $\frac{c^5}{G}$ (Kostro 2010, pp. 165-179). Aristotle and medieval philosophers talked already about *minima et maxima naturalia.* F_{lim} and P_{lim} are *maxima naturalia.*

2. The players of the proposed toy-model

Let's introduce first the players that I will use frequently in the Λ-toy- model of our visible universe and its Dark Energy.

a. c – the limitary velocity of transfer of energy and momentum. The velocity of light is the best example of such a limitary transfer.
b. G – gravitational mathematical coefficient introduced by Newton to connect the gravitational force $F = mg$ with two gravitationally interacting charges m and m inversely to the square of distance $\frac{1}{R^2}$, $\quad F = mg = \frac{Gm^2}{R^2}$. The ratio $\frac{m^2}{R^2}$ has not the dimension of a force. To receive such dimension we must multiply it with the coefficient G.

Note, however, that its inverse $\frac{1}{G}$ = 1. 498 · 10^{10} kg m^{-3} s^2 has a specific physical meaning in GR. The dimensional analysis of it shows that it is a product of mass density $\varrho = \frac{m}{l^3}$ and the square of time t^2. Hence $\frac{1}{G} = \frac{m}{l^3}t^2 = \varrho t^2$. For example, in the introduced above Stoney's, Planck's and Kittel's Units we are dealing with the following relations:

$$\varrho_p t^2_p = \varrho_s t^2_s = \varrho_G t^2_G = \frac{1}{G} = 1.\,498 \cdot 10^{10}\ kg\ m^{-3}\ s^2$$

Note that the relation between density and time has also a real physical meaning in the whole Universe shown in Friedman's equations.

c. The physical cosmic meaning of the inverse of the Newtonian gravitational constant $\frac{1}{G}$.

Since the gravitational constant concerns the whole universe let's study the part played by the relation mass density and square time $\frac{1}{G} = \frac{m}{l^3}t^2 = \varrho t^2$ in the standard model of the Universe in which it is assumed that $\Lambda>0$ and its geometry is flat and therefore its curvature parameter $k = 0$. Let's repeat that the relation ϱt^2 is contained as a basic mathematical structure in Friedman equations also in the standard model in which on the base of recent observations and theoretical consideration the age of the Universe is estimated to be $T_{H_0} = \frac{1}{H_0}$ ~ 13,84 billion years i.e,~ $4.367452262 \cdot 10^{17}$ s.

The Friedman equation of the model with $k = 0$ and $\Lambda > 0$ has the following form

$$(8 \pi G \varrho_G /3) + (\Lambda c^2/3) - H^2 = 0$$

(where ϱ_G is the average mass density of the gravitational ponderable matter).

In the standard model the average mass density $\varrho = \varrho_G + \varrho_\Lambda$ in the universe is considered to be equal to the so-called time depended critical density ϱ_{crit} . It means that

$$\varrho = \varrho_{crit} = \frac{3H_0^2}{8\pi G} \text{ and } = \frac{3}{8\pi G T_0^2}$$

because in the standard model the Hubble parameter $H_0 = \frac{1}{T_{H_0}}$. The average mass density is considered as the sum of the average mass density of the gravitational ponderable matter ϱ_G and the mass density ϱ_Λ of lambda mass corresponding to Dark Energy.

$$\varrho = \varrho_G + \varrho_\Lambda = \varrho_{crit} = \frac{3H_0^2}{8\pi G} = \frac{3}{8\pi G T_0^2} \quad \sim 9{,}46 \times 10^{-27}\ kg/m^3$$

Remember that we use in the considered model also the dimensionless density parameters $\Omega_G = \varrho_G / \varrho_{crit}$; $\Omega_\Lambda = \varrho_\Lambda / \varrho_{crit}$ related between them as follows $\Omega_G + \Omega_\Lambda = 1$. On the basis of observational WMAP 7-year results $\Omega_\Lambda = 0{,}728 \pm 0{,}016$ and $\Omega_G = 0{,}272 \pm 0{,}016$. It means that in our present day universe we are dealing with about 72% of Dark Energy and about 28% of gravitational ponderable matter.

Which is the relation between the present day critical density ϱ_{crit}, and the present day square age of our Universe T_0^2 ? Is it equal to $\frac{1}{G}$? The $\frac{1}{G}$ enters in the relation but with a dimensionless coefficient. Here it is: $\varrho_{crit}\ T_0^2 = (\frac{3}{8\pi})\ \frac{1}{G}$; The dimensionless coefficients for $\varrho_G T_0^2$, $\varrho_\Lambda T_0^2$ and also $\varrho_\Lambda t_\Lambda^2$ have their own dimensionless coefficients used in GR. When the age of the universe was equal to t_Λ then the coefficient was $\left(\frac{1}{8\pi}\right)$. Therefore we write $\varrho_\Lambda t_\Lambda^2 = (\frac{1}{8\pi})\ \frac{1}{G}$.

d. $\kappa = 8\pi G c^{-4}$

The constant κ is sometimes called Einstein's gravitational constant. It is the coefficient that is a component of his General Relativity equation. It has the dimension of an inverse force, as was already indicated, of the lambda force

$$F_\Lambda = c^4/8\pi G = \frac{1}{\kappa} = 4{,}81 \cdot 10^{42}\ N$$

As well known in GR we do not speak about gravitational force. Gravitation is considered as result of the curvature of space – time. However the GR equation that contains in itself the force $\frac{1}{\kappa}$ in a hidden way is not complete. When we would like to apply it to the whole universe there must be introduced into it a centrifugal pressure to avoid the gravitational collapse. This pressure consists in the acting of the lambda force $\frac{1}{\kappa}$ on an inverse surface that is just lambda *Λ*. So we see that the force $\frac{1}{\kappa}$ is not a gravitational force but the *Λ*-force. It is present in the action Lagrangian $S = \int [\frac{1}{2\kappa}(R - 2\Lambda) + \mathcal{L}_M]\sqrt{-g d^4 x}$.
Note that from this Lagrangian can be derived the Einstein's equation of GR with *Λ* applied by him to the whole universe.

e. *Λ* constant.

When Einstein in 1917 applied his General Relativity to the whole Universe he became aware that because of gravitation his universe will collapse. But he was convinced that the universe is stable. Therefore to introduce equilibrium and stability he introduced an anti-gravitational pressure of the vacuum. We are dealing with pressure when a force is acting on a surface. In our case it is the lambda force $\frac{1}{\kappa}$ acting on lambda surface A_Λ

$$P_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa} \sim 5{,}73 \cdot 10^{-10}\ N \cdot m^{-2}$$

In our human scale it is weak. Some properties of the Dark Energy are considered by cosmologists as constant e.g. its density of mass and energy.
The lambda pressure P_Λ is known since long time in Relativistic Cosmology, but the majority of lambda units that I try to introduce are unknown.
As regards the cosmological constant *Λ*. it has the dimension of an inverse surface i.e. a unit surface on which the *Λ* - force $= \frac{1}{\kappa}$ presses. We can obtain it for the standard model as result of Friedman eqs.

$$\Lambda = 3\Omega_{\Lambda o}/R^2_{Ho} = 3\Omega_{\Lambda o}/c^2 T^2_{Ho} \quad \sim 1.28 \cdot 10^{-52}\ m^{-2}$$

Where R_{Ho} is the present day Hubble radius and T_{Ho} - the present day Hubble time i.e. the present day age of the Universe.

3. The three basic lambda units: Lambda-length, Lambda-time and Lambda-mass reminded

Since *Λ* has the dimensions of an inverse surface we obtain, in easy way. the *Λ* - surface. Inverting *Λ* we obtain

$$\Lambda\text{ - surface} \qquad A_\Lambda = \frac{1}{\Lambda} \sim 7{,}8 \cdot 10^{51} m^2$$

Since A_Λ as unit is a square the *Λ* - length is its side. Hence

$$\Lambda \text{ - } length \qquad l_\Lambda = \frac{1}{\Lambda^{1/2}} \sim 8{,}84 \cdot 10^{25}\, m \ \sim \ 9{,}34 \text{ billion light years}$$

Since $l_\Lambda = ct_\Lambda$ hence $\quad t_\Lambda = \frac{1}{c\Lambda^{1/2}} \sim 2{,}95 \cdot 10^{17} s \sim$ 9,34 billion years

As we can see l_Λ and t_Λ are extremely large especially with respect to Stoney's l_s and t_s and Planck's l_P and t_P which are extremely small.

Λ - mass as unit can be derived in simple way writing the product of Λ - mass density ϱ_Λ (known since longtime in the standard model) and of Λ - volume as unit ${l_\Lambda}^3$

$$m_\Lambda = \varrho_\Lambda {l_\Lambda}^3 = \frac{c^2}{8\pi G \Lambda^{1/2}} = \frac{1}{c^2 \kappa \Lambda^{1/2}} \sim 4.75 \cdot 10^{51}\, kg$$

In the same way we derive the unit Λ - energy E_Λ writing the product of Λ - energy density $\varrho_\Lambda c^2$ (known also since longtime in the standard model) and of Λ -volume as unit ${l_\Lambda}^3$

$$E_\Lambda = \varrho_\Lambda c^2 {l_\Lambda}^3 = \frac{c^4}{8\pi G \Lambda^{1/2}} = \frac{1}{\kappa \Lambda^{1/2}} \sim 4{,}27 \cdot 10^{68}\, J$$

They are also extremely large especially with respect to Stoney and Planck mass and energy. In my paper (Kostro 2018) I introduced the whole list of Λ - Units.

Among the Λ - units there are (1) the Λ - Mega Quantum of Action

$$H_\Lambda = \frac{1}{c\kappa\Lambda} \sim 1.35 \cdot 10^{86}\, J \cdot s. \sim 8.42 \cdot 10^{95}\, GeV \cdot s$$

and (2) the Λ - Mega Quantum of Angular Momentum

$$'H_\Lambda = \frac{1}{2\pi c\kappa\Lambda} \sim 2{,}15 \cdot 10^{85}\, J \cdot s\,.$$

They look esthetically very well because in them there is an inverse of the three Einstein's constants *c, κ* and *Λ*. But have they any physical meaning? Does correspond to them anything in the physical reality? Can we attribute a kind of spin to all visible spheres? Let's now put them in their historical context.

4. Some historical data concerning the introduction and physical meaning of the quantity called action

The physical quantity called *action* has been introduced into classical physics by Pierre Louis Moreau. de Maupertuis (1698-1759). He also introduced the principle of the least action. In order to understand the physical meaning of the new quantity let me first introduce some historical data concerning the evolution of the dynamical conception of the physical causality in classical physics and its impact on modern physics. To visualize the evolution let's use the following schedule

	cause		effect
	cause		effect
Newton	F	=	ma
Descartes	$F\Delta t$	=	$m\,\Delta v$
Leibniz	$F\Delta l$	=	$\frac{mv^2}{2}$
Maupertuis	$F\Delta l\,\Delta t$	=	$ma\,\Delta l\,\Delta t$
	$W\Delta t$	=	$E_{kin}\,\Delta t$
	$I\Delta l$	=	$m\,\Delta v\,\Delta l$

According to Newton the category of cause is represented by the force F itself and the category of effect by acceleration a. Descartes was of the opinion that the category of cause is represented by the impulse of the force $F\Delta t$ because according to him the force to be able to do something must act during certain time interval and the effect manifests itself in the increase of momentum $m\,\Delta v$, Leibniz was of the opinion that the category of cause is represented by the work $F\Delta l$ done by force because according to him the force to be able to do something must act along certain path and the effect manifests itself in the increase of kinetical energy $\frac{mv^2}{2}$. Finally Maupertuis has shown that the acting force in order to do something must act along certain path and during certain time. So in his definition of the physical cause we are dealing with the acting force that has its path and time of acting. He gave to $F\Delta l\,\Delta t = W\Delta t = I\Delta l$ the name «action».

We are dealing with action not only in an accelerated motion considered above but also in all other kinds of motion because in all kinds of movement we are dealing with a transfer of momentum and energy along certain spatial distance (path) and during certain time (time interval) and also with a transfer of angular momentum in rotational motion when the angle of rotation changes.

The known Polish physicist Czesław Białobrzeski has often pointed out that action is the richest in meaning physical quantity because the notion of action expresses a physical dynamical process, in which the dynamical quantities are connected with space-time parameters:

		dynamical quantities		space-time parameters
action	=	*momentum*	×	*path*
action	=	*energy*	×	*time*
action	=	*angular momentum*	×	*angle of translation*
action	=	*force*	×	*path × time*
action	=	*work*	×	*time*
action	=	*impulse of force*	×	*path*

At the beginning of XX century the quantic nature of action and angular momentum in the micro-world was discovered. This quanta are interpreted sometimes as fundamental physical events because we deal in such cases with quantic transfer of energy and momentum along a certain distance and during certain time interval and also with a quantized transfer of angular momentum in rotational motion when the angle of rotation changes.

Maupertuis formulated also the "Least Action Principle" describing the trajectory in space and time of the transfer of energy and momentum However, when the variational calculus has been introduced into the formulation of the mentioned principle it has been discovered that action is submitted to a larger variational principle because it is not only a minimum but, in certain cases, it can be also a maximum. Therefore it is now more correctly to call it principle of extremal action. It is often called also principle of stationary action. Since the Planck's quanta of action and angular momentum h and $\hbar$ and Stoney's quantum of action h_S, which govern in the micro-world, are extremely small (h = 6.626 · 10^{-34} J · s ; $\hbar = \frac{h}{2\pi}$.= 1.054 · 10^{-34} J · s and $h_S = K\frac{e^2}{c}$ = 7,69 · 10^{-37} $J \cdot s$) we can ask the question whether the Λ- Mega Unit of Action, which is extremely Large ($H_\Lambda = \frac{1}{c\kappa\Lambda} \sim 1.35 \cdot 10^{86}$ $J\cdot s$ $\sim 8.42 \cdot 10^{95}$ $GeV\cdot s$), is the Mega Quantum of Action which governs in the Large Scale–World.

The stationary (extremal) principle of action mentioned above was central in the classical physics and remains central in modern physics being applied in the theory of relativity (special and general), in quantum mechanics and quantum field theory.

In General Relativity applied to the cosmos i.e. with Einstein's cosmological constant Λ the Lagrangian of action has the following form.

$$S = \int [\frac{1}{2\kappa}(R \; -2\Lambda) + \mathcal{L}_M] \sqrt{-g d^4} x$$

Where $\frac{1}{\kappa}$ is the lambda force, R is the Ricci curvature scalar, Λ is Einstein's cosmological constant, the term $\mathcal{L}_M$ is describing any matter fields appearing in the theory and g = det ($g_{\mu\nu}$) is the determinant of the metric tensor.

When we introduce the lambda unit of action into the action Lagrangian of GR then it looks as follows

$$S = \int [H_\Lambda (R\frac{\Lambda c}{2} - \Lambda^2 c) + \mathcal{L}_M] \sqrt{-g d^4} x$$

In such a way we can see how the action in General Relativity with Λ depends in a natural way on the Λ - mega unit of action. This aspect is here explicitly shown.

5. Toy-point-observers

Like Boltzman has introduced his demon and Schrödinger his cat let me introduce the toy-point- observers that exist in every point of our universe from the beginning and have eyes looking in all directions. Each of them has his sphere of observation, his visible sphere. The background radiation is seen in each visible sphere as their visible horizon

Their visible universes are a spherical volumes (balls) centered on a toy-observer. Every location of a toy observer in the Universe has its own visible universe, which may or may not overlap with the one centered on Earth. The radius R_o of each visible sphere increases with time. The present day R_o is for each observer about 13, 84 billion light years long. Each visible sphere is a ball causally bounded because of the limitary velocity *c.* The horizon of each toy observer runs away with the velocity *c*. In the expanding space of the Universe the causally bounded observation spheres can be separated, can touch or partially cover (overlap) each other.

Cosmologists has introduced also another observable sphere called Hubble sphere in which it is shown how far the observed galaxies are really now. The edge of the observable universe conceived in such a way is about 46,5 billion light years away. So the diameter of the Hubble sphere is about 93 billion light-years long.

According to our present day knowledge our universe is flat at least in the scale of our visible sphere and therefore we can use with great approximation to such a sphere the Euclidian equation $V = (4/3)\pi\, R^3$. So, when the age of the universe was equal to the Λ - time the Λ - sphere volume was $S_\Lambda = \frac{4\pi}{3(\Lambda^{1/2})^3} \sim 3{,}22 \cdot 10^{78} m^3$.

When we "turn back our clocks" then we can imagine the past. The visible spheres would become smaller and smaller and the toy-point-observers would be closer and closer to each other. But there are minimal limitary distances between them. In Planck system of units the Planck length is such a limitary distance.

6. Crucial age in the evolution of our universe. The new acceleration of its expansion called by some cosmologist the second inflation that took place about 9 billion years after the Bing Bang does it coincides with the lambda time?

Every point in the universe (from which the universe becomes "visible" when we connect with it "a toy-point-observer") is the center of the centrifugal pressure that is the source of the expansion of the universe in all directions. The accelerating expansion of the universe manifests itself in the observation that the universe expands at an increasing rate, so that the velocity at which a distant cluster of galaxies is receding from the observer is continuously increasing with time. The accelerated expansion was discovered in 1998, Cosmologists at the time expected that the expansion would be decelerating due to the gravitational attraction of the matter in the universe. The observations have proved the opposite. It is estimated that the second inflation began about 9 billion years after the Bing Bang. So the expansion of the universe is thought to have been accelerating since the universe entered its dark-energy-dominated era roughly 5 billion years ago.

It is interesting to note that just at the crucial age the radius of the universe was about 9, 34 billion light years long (i.e. it was equal to the $\Lambda -$ length $l_\Lambda = \frac{1}{\Lambda^{½}} \sim 8{,}84 \cdot 10^{25}\ m$) and the age of the universe was about 9, 34 billion years old (i.e. it was equal to Λ - time $t_\Lambda = \frac{1}{c\Lambda^{½}} \sim 2{,}95 \cdot 10^{17} s$. Let's add that in the crucial event Λ - time became fully real and the $\Lambda -$ length as well. Thus we can say that the age of the accelerating expansion of the universe and the Λ - time coincide. At the crucial age of the universe $T_+ = t_\Lambda = \frac{1}{c\Lambda^{½}}$ also the Mega Quantum of Action $H_\Lambda = \frac{1}{c\kappa\Lambda}$ fully occurred. The lambda force $\frac{1}{\kappa}$ acting in all directions along the crucial radius of the causally bounded visible universe $R_+ = l_\Lambda = \frac{1}{\Lambda^{½}}$ during the crucial age of the universe $T_+ = t_\Lambda = \frac{1}{c\Lambda^{½}}$ does perform the Mega quantum of Action $H_\Lambda = \frac{1}{\kappa} \cdot \frac{1}{\Lambda^{½}} \cdot \frac{1}{c\Lambda^{½}} = \frac{1}{c\kappa\Lambda} {\sim} 1.35 \cdot 10^{86}\ J \cdot s$). If we assume the interpretation that quanta of action have to be considered as physical events then at the beginning of the crucial era a Mega Physical Event happened. At the crucial event there appeared also the lambda portions of energy as unites $E_\Lambda = \varrho_\Lambda c^2 {l_\Lambda}^3 = \frac{c^4}{8\pi G \Lambda^{½}} = \frac{1}{\kappa \Lambda^{½}} \sim 4{,}27 \cdot 10^{68}\ J$ (i.e. lambda energy contained in a *Λ* - volume forming an Euclidian cube with the side equal to $l_\Lambda = \frac{1}{\Lambda^{½}}$) and the portions of the corresponding *Λ* - mass units $m_\Lambda = \varrho_\Lambda {l_\Lambda}^3 = \frac{c^2}{8\pi G \Lambda^{½}} = \frac{1}{c^2 \kappa \Lambda^{½}} \sim 4.75 \cdot 10^{51}\ kg$ contained in the same *Λ* - volumes.

6.1. Which was the proportion between the Dark Energy and Gravitational Ponderable Matter at the crucial age $T_+ = t_\Lambda$ of the Universe?

At the crucial age $T_+ = t_\Lambda = \frac{1}{c\Lambda^{½}}$ the so-called critical density of the Universe was $\varrho_{crit+} = \frac{3H_+^2}{8\pi G} = \frac{3}{8\pi G T_+^2} = \frac{3}{8\pi G t_\Lambda^2}$. Then the a-dimensional density parameters $\Omega_{\Lambda+} = \varrho_\Lambda / \varrho_{crit+} = 0{,}33$ and $\Omega_{G+} = \varrho_G / \varrho_{crit+} = 0{,}66$. It means that at the crucial age of the Universe there were 33% of Dark Energy and 66% of Gravitational Ponderable Matter.

Nevertheless the centrifugal pressure of Dark Energy overcame the gravitational attraction of the ponderable matter. Why? The gravitational interactions are interactions between masses of the ponderable matter. So we can assume that at the crucial age the half part of the ponderable matter interacted with the second part. We can divide the matter inside the causally bounded sphere doing symmetrical sections of the sphere into two hemispheres, Such hemispheres interact on the average at the distance equal to the radius of the sphere $R_+ = l_\Lambda = \frac{1}{\Lambda^{½}}$. At the crucial age the Kittel's gravitational action, is given by $H_{G+} = \frac{G {m_{G+}}^2}{c}$ where m_{G+} is the half part of M_{G+}. Simple mathematical transformations and calculations show that $H_{G+} = \frac{H_\Lambda}{8\pi} = \frac{1}{8\pi c \kappa \Lambda}$ and that, applying the Newtonian approximation, the $F_{G+} = \frac{F_\Lambda}{8\pi} = \frac{1}{8\pi\kappa}$ As we can see the gravitational interactions began then to be 8π times weaker than the *Λ* - pressure and so the second inflation could develop. Note that the quantity of Dark energy increases when the radius *R* of the causally bounded sphere increases. The radius is raised to the third power according to the equation $E_{\Lambda o} = \varrho_\Lambda c^2 {R_o}^3$.

6.2. Will there be a second crucial age in the evolution of our Universe when it enters in the age equal to 2 t_Λ i.e. when our Universe will be 18,68 billion years old? Will then begin a second crucial epoch T_{++} ?

On the one hand when the lambda epoch like era began the quantity of the Dark Energy permanently increases. On the other hand the gravitational ponderable matter becomes permanently rarified and therefore the gravitation attraction is becoming weaker and weaker. What will be happen when our Universe will enter in the age $T_{++} = 2t_\Lambda = \frac{2}{c\Lambda^{½}}$ ~18,68 billion years? Will it change its topology? Now in the standard model (i.e. with the curvature parameter k=0 and Λ>0) it is considered to be flat. Will it change its curvature parameter into k=+1 or k =-1? These are open questions. Note that at the second crucial age $T_{++} = 2t_\Lambda = \frac{2}{c\Lambda^{½}}$ the so-called critical density of the Universe will be $\varrho_{crit++} = \frac{3H_{++}^2}{8\pi G} = \frac{3}{8\pi G T_{++}^2} = \frac{3}{8\pi G (2t_\Lambda)^2} = \frac{3}{\kappa c^4 (2t_\Lambda)^2}$. Since $(2t_\Lambda)^2 = \frac{4}{c^2\Lambda}$ we obtain finally $\varrho_{crit++} = \frac{3\Lambda c^2}{4\times 8\pi G} = \frac{3\Lambda}{4\kappa c^2}$. The mass density of Dark Energy is treated in the standard model as constant $\varrho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{\Lambda}{\kappa c^2} = \text{const}$. Let's now calculate the a-dimensional density parameter $\Omega_\Lambda = \varrho_\Lambda / \varrho_{crit++} = \frac{\frac{\Lambda}{\kappa c^2}}{\frac{3\Lambda}{4\kappa c^2}} = \frac{4}{3}$ = 1,33. As we can see ϱ_Λ is greater than ϱ_{crit++}. What does it mean? I ask the question to the scientific community.

Let's conclude Stoney's and Planck units play an important part especially at the beginning of the Universe instead the lambda units in its evolution that is still investigated.

7. The proposed toy-model fits to a certain extent with Louis de Broglie relativistic wave mechanics that was introduced at the historical beginning of QM

Perhaps we have to start with de Broglie wave mechanics of large scale. De Broglie started with the assumption that $h\nu = mc^2$. Let's follow him taking into account the Λ - Units.

$H_\Lambda \nu_\Lambda = m_\Lambda c^2 = E_\Lambda \sim 4{,}41 \cdot 10^{68}\ J$ where $\nu_\Lambda = \frac{1}{t_\Lambda}$ (i.e. ones every 9.34 billion years) and the lambda wavelength is then equal to lambda length $\lambda_\Lambda = l_\Lambda = \frac{H_\Lambda}{m_\Lambda c} \sim$ 9,34 billion light years and the lambda wave period is equal to lambda time $T_\Lambda = t_\Lambda = \frac{H_\Lambda}{E_\Lambda}$ ~ 9,34 billion years.

Let's repeat that the lambda mass M_Λ is not a ponderable gravitational mass. The Dark Energy is a pure energy and its corresponding mass is not a rest mass. The lambda wave can be treated by the toy-observers in their visible spheres as a spherical wave the front of which is running away with the velocity c . Its frequency $\nu_\Lambda = 1/\ T_\Lambda = c\Lambda^{½} = (c^2\Lambda)^{½} \sim 3{,}27 \cdot 10^{-18} s^{-1}$.

Let's return to Maupertuis operational definitions of the physical dynamical cause considered as physical action: $F \cdot \Delta l \cdot \Delta t$ (i.e. product of the acting force and path and time of its acting) or $W \cdot \Delta t$ (*work · time*) or $I \cdot \Delta l$ (*impulse of force · path*).

Let's now apply them to the causality of Dark Energy. Its repulsive action caused by the centrifugal lambda force $F_\Lambda = \frac{1}{\kappa}$ in all directions along the radius $R_+ = l_\Lambda$ and during the time $T_+ = l_\Lambda$ of expansion was with respect to each chosen toy- point observer in our Universe when our Universe was 9,34 billion years old. Then in every causally bounded observational sphere a lambda quantum of action H_Λ occurred. There (i.e. in every causally bounded observational sphere) occurred also a mega quantum of centripetal action of the ponderable matter $H_{G+} = \frac{H_\Lambda}{8\pi} \sim 5.37 \cdot 10^{84}\ J \cdot s \sim 3.35 \cdot 10^{94}$ GeV· s. So when the first crucial epoch happen two mega quanta occurred in the visible causally bounded spheres in the Universe. In the micro-world there are working the Planck's and Stoney's quanta of action which are extremely small. Perhaps in the large-scale-world there are working large-scale quanta of action which are extremely large.

Since in each visible sphere the distribution of the ponderable matter (Dark Matter and Ordinary Matter) is very random and lambda stands in relation with the metric tensor Λg, with its components ($g_{\mu\nu}$) therefore we can try to introduce Mega Heisenberg uncertainties relations concerning Dark Energy and Ponderable Matter. That is only my suggestion

$$\Delta\, E_\Lambda\, \Delta\, t_\Lambda \geq H_\Lambda \qquad \Delta\, E_G\, \Delta\, t_G \geq H_G$$
$$\Delta\, p_\Lambda\, \Delta\, l_\Lambda \geq H_\Lambda \qquad \Delta\, p_G\, \Delta\, l_G \geq H_G$$

Can H_G that occured also at the crucial moment be considered as constant or is it time dependent because the density of ponderable matter slows down. Both of them occurred at the critical crucial moment. Can the two constants H_Λ and H_G become paramiters of the future quantisation in mega scale? I am not able to answer this question? That is an open question that I ask the scientific community. When the answer will be "Yes" then we can try to introduce the Heisenberg like mega uncertainty relations. At the crucial moment the first lambda mega quantum of action H_Λ occured. The density of the ponderable matter mass and energy began to decrease. Perhaps the unification of the Relativistic Cosmology with QM has to be done in the mega scale domain and not in the micro scale one.

8. Final remarks

Gravitational and lambda interactions are entirely negligible and insignificant in the micro world of the elementary particles and at the beginning of the universe evolution. But they are very significant in the mega scale among celestial bodies, galaxies and especially among the clusters of galaxies and therefore H_Λ and H_G play their part in Mega Scale. Note that the gravitational waves were discovered when great black holes collided. Perhaps lambda fluctuations will be discovered in large scale phenomena like e.g. the so-called second inflation. When the age of the universe will arrive to $2t_\Lambda$ what will happen? The future generations will answer the question.

We must be aware that mathematical models of reality are not the reality but only our mental constructions that help us in our cognition of the reality. The mathematical model of the atom of hydrogen is not an atom of hydrogen as well as the mathematical model of our universe is not the universe.

All our mathematical models contain approximations and sometimes dangerous simplifications. We must control and investigate the degree of approximation and simplification. We must control if the degree of approximation and simplification are still admissible and therefore good or bad. In a good theory the degree of approximation should be the highest possible and the degree of the bad simplification the lowest possible.

Perhaps the introduced in my paper toy-model belongs to models created only by my human fantasy and Einstein if he was present at our congress he would get a loud laugh of it.

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