The role of mathematicians in the development of early science. A new insight

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Abstract: "Science is mathematics". This proposition is contemporaneously false and true. Establishing its truthiness depends on the meaning given to the terms *science* and *mathematics*. If they are considered as historical categories, the proposition is clearly false. Today mathematics is considered as distinct from science; the former being essentially rational, the latter making necessarily recourse to experience. In the past mathematics had a more variegate meaning than today. It comprehended some parts that we can call science – modern meaning; for instance, mechanics, astronomy. Science was instead only a theoretical discipline, not necessarily related to experience and thus differed from mathematics. "Science is mathematics" may become true if mathematics is understood with its ancient meaning and science with its modern meaning. Notice, however, that there are sciences (modern meaning) that are not mathematics (ancient meaning). They are, for instance: linguistics, anatomy, botanic and other disciplines that make a limited recourse to the category of quantity. They can become mathematics only if the meaning of mathematics is enlarged to include logic.

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1. Mathematicians and science

For mathematician I mean the scholar who knew mathematics, in the modern sense, but that also was busied by other; first of all applied mathematics or mixed mathematics – with a Renaissance terminology – that is music, optics, astronomy and mechanics. But he could also take care of medicine, architecture, etc. Mathematician, in this sense, is a term used from 1500 to 1700. In this sense, modern engineers, physicists, chemists and mathematicians can be qualified as mathematicians.

With a strong enough expression, I hope not trivial, it can be said that mathematicians were by far the proponents of science, as we know it today. At least for the part that goes under the name of exact science. The role of professional philosophers – not of philosophy in a broad meaning –, especially the philosophers of nature, was certainly important, but much less (Capecchi 2017).

Many of the problems posed by mathematicians, as nature of the continuum, existence of vacuum, driving force, gravity and motion, were also studied by the professional philosophers of Nature. But they were essentially general problems posed to any educated person. I mean that the metaphysical and epistemological problems were not exclusive domain of professional philosophers. Mathematicians, from classical and Hellenistic Greece, had their own ideas about many metaphysical problems of philosophy that were dealt with differently from the professional philosophers. Many mathematicians could have little philosophical knowledge as modern scientists do, but this notwithstanding they thought about Nature. The so-called Pythagoreanism, that is the attribution of a fundamental role of mathematics in the interpretation of the world, usually rooted in the influence of Pythagoras and Plato, is a constituent of Greek mathematics and precedes the two philosophers.

To get an idea of the origin of modern science, it is particularly instructive to look at the applied mathematics of ancient Greece, and in particular of the Hellenistic period. Greek mathematics was born at the same time as astronomy, harmonics, mechanics, optics and surveying. Only later began a process of abstraction that eliminated, but not completely, the sensitive basis, thus separating pure from impure mathematics (contaminated by the senses). Impure, or "applied", mathematics, or mixed mathematics as referred to in this paper, continued to exist and were generally studied by the same scholars who dealt with pure mathematics. This without a sharp distinction of roles and status between them. Mathematicians, indeed, generally were not only specialists, that is they were not mathematicians in the modern sense of the term; many of them shared interest in natural philosophy, epistemology, technology, medicine. They were thus able to develop ideas about the nature of the world independently enough of those of "professional" philosophers and theologians. They went some way to build a community with shared values; they knew each other both diachronically and synchronically, criticizing or esteeming, but in any case commenting on each others' works. This community pursued its science not only out of a love of knowledge, not only to know the fact and the reasoned fact as philosophers did, with the aim to make predictions, which only allowed the improvement of technology.

At least since Ptolemy (II century AD), astronomy and music were characterized by a hypothetical deductive approach, with theories that were not validated directly – which was impossible – but according to their observable consequences (Capecchi 2014, 2016). Based on a preliminary examination of a phenomenon he proposed a mathematical theory. It was then placed in relation with the experimental data that could be obtained with laboratory experiments, or with accurate observation thanks to special instrumentation. This is particularly clear in the writings of music theory where Ptolemy's epistemology was made explicit.

It is clear that a careful reading of texts with their contextualization in the period made manifest differences – virtually impossible to make precise – with respect to the modern approach. I know quite well the criticism in a modernist interpretation of the hypothetical deductive method, but consider that yet today it is difficult to give a unique definition of what a hypothetical deductive method is for a scientist.

Here is what Ptolemy wrote in his *Harmonica*:

> The purpose of the harmonicist would be then to preserve in every way the reasoned hypotheses of the canon which do not in any way at all conflict with the perceptions

> as most people interpret them, just as the purpose of the astrologer is to preserve the hypotheses of the heavenly movements concordant with observable paths. Even these hypotheses are themselves assumed from what is clear and roughly apparent, but with the help of reason discover detail as much accuracy as is possible (Ptolemy 2000, pp. 7-8).

Optics had a simpler structure based either on empirical principles verified by controlled experiments, or they are self-evident. Here is what Ptolemy wrote in his *Optics*:

> For all cases in which scientific knowledge is sought, certain general principles are necessary, so that postulates that are sure and indubitable in terms either of empirical fact or of logical consistency may be proposed and subsequent demonstrations may be derived from them. We should therefore indicate that three particular principles are needed for the scientific study of mirrors and that, being of the first order of knowledge, they can be understood by themselves (Ptolemy 1996, p. 131).

The situation of mechanics, prince discipline of physics at least for a long time, was very special. Mechanics is at the same time the farthest from mathematics and the nearest. It is the farthest because it refers to concepts such as weight, strength (apart from mass and inertia) that cannot be translated into geometrical concepts. It is the closest because its principles are so obvious that they can be accepted by all, as the principles of Euclidean geometry. To develop a general theory of mechanics, it is not necessary to perform experiments. In particular, the principles of statics, the law of the lever and the rule of the parallelogram, can be justified by referring to common sense, as that stating that if a stone is left free, it falls downwards.

Euclid’s and Archimedes’ mechanical texts are particularly enlightening to understand the mechanics theory at its beginning. The equilibrium of solids is reduced to the determination of the centers of gravity with Archimedes. The only (or almost) empirical postulate is that saying that if on a scale with equal arms two different weights are hung, the scale tilts toward the heavier. In the discussion on the equilibrium of fluids, the situation is a bit more complex; here concepts are used such as force and pressure, necessarily endowed with mechanical meaning.

Mixed mathematics crossed the Middle Ages with important changes but without altering the substance. Astronomy, optics, music and mechanics underwent a change by an interaction with new natural philosophy, that became strong in the XVII century, the abrupt development of technology and the recovery of ancient mathematics. The new mathematics of Renaissance – algebra – played some role too, but much remains to be studied about it. A factor that favored these changes was the diffusion of printing. In ancient Greece an author, even referring to an extant treatise, could not assume that his reader was acquainted with it. Thus instead of dealing only with advancement he dealt also with the argument of the cited treatise, by treating it in a more thorough way. This determined a mechanism of self-repetition, rather than of evolution. The diffusion of printing allowed to break this circularity.

Astronomy in the Middle Ages was essentially a geometrical discipline with only the aim to save phenomena and no claims were raised about causal explanations. First with Copernicus and then with Kepler, it became again a “physical” discipline, at least from what concerned the solar system. Optics changed from a theory of vision to a theory of light transmission; music moved toward acoustics, and mechanics gave raise to dynamics (modern term), that is a mathematical science of motion. Mechanics, that in the Middle Ages had become the science of weights, concentrate on the scale, became mechanics again in Greek meaning, as science of machines.

The interaction of mathematics with physics was restricted to traditional mixed mathematics and some other disciplines close to them, such as surveying, architecture, ballistics. For other disciplines, traditionally fully framed into the natural philosophy, mainly based on experience and experiment, such as magnetism, electricity, thermology, alchemy/chemistry, biology, physiology etc., the role of mathematics was different and the interaction slower. What was taken from mathematics was the way of reasoning; that is the use of clear definitions, the rejection of the use of synonymous and homonymous, assumptions derived from experiments and considered as true; the use of a deductive approach for proving propositions, even without the explicit use of geometry or arithmetic. For some sciences the “evolution” toward a form of mixed mathematics, started partially in the XVII century, lasted at least until the XIX century; this was the case of disciplines founded on quantitative descriptions such as for instance magnetism, electricity, chemistry. Other sciences, where the use of quantity was negligible, such as structural botany and zoology, philology, morphology, that could be classified as qualitative sciences, did not reach, and until today have not yet reached, the status of mixed mathematics. For them, the use of symbolic logic however allowed, and yet allows, at least in principle, an approach that has a similar deductive structure of that of mixed mathematics.

A good enough idea of the evolution of (mixed) mathematics toward modern science can be reached considering in detail the evolution of mechanics, that often, at least in the past, was considered the prototype for understanding the scientific “revolution”. For the sake of space, reference has been made mainly to the period close to Galileo who was a main character in the field of mechanics at the turn of the XVII century.

Mechanics until the XVII century was the name of the (mixed) mathematics that took care of the functioning of simple machines (lever, block and tackle, winch, screw, wedge), and their combinations thereof. In ancient times it reached a peak with Hero of Alexandria in the I century AD. It was a discipline strongly mathematized that had, at its basis, concepts of empirical character, but whose evidence or acceptance was immediate to the point that it was sometimes considered a purely rational discipline. Foundation of mechanics was the law of lever.

There were two distinct justifications of this law:

1. The Archimedean one, based on symmetry considerations and absolutely certain empirical statements, such as: if to a scale with equal arms are suspended two weights, the scale tilts on the side of the greater weight.

2. The one called, quite improperly, Aristotelian. It had a kinematic character; the equilibrium is due both to weights and their virtual motions (virtual work law).

Hero's Hellenistic mechanics allowed to solve all the problems of equilibrium, however complex, even if its application required a certain ingenuity in reducing all the mechanisms to the lever. The actual occurrence of equilibrium for a system established by the law of the lever left no room for doubt. An experimental test, besides not being considered necessary, even seemed inconceivable, at least within a certain limit.

In the early modern era, with a new mature mathematics, the laws of lever and virtual work gave raise to more effective tools, such as for example the law of the parallelogram of forces; first with Leonardo da Vinci, then with Simon Stevin and Gilles Personne de Roberval. Mechanics, however, still remained essentially a geometric discipline.

Things changed when besides equilibrium, mathematicians set themselves the objective of studying the motion or, using modern terminology, they began to deal with dynamics. A development that was natural when one thinks as the machines as essential tools for moving weights. In fact, in the early XVII century the term *mechanics* spread to indicate the integrated science of statics and dynamics.

The science of motion, since antiquity had been a fundamental and exclusive part of natural philosophy, especially that of Aristotle, who recognized four types of motion or changes, with local motion coinciding with our "vulgar" concept. There is a coincidence, hardly by chance, between the birth of the science of motion and the spread of artillery. With Niccolò Tartaglia ballistics was born, a (mixed) mathematics (Tartaglia 1537) which studied the motion of a heavy mass point (a bullet). But only with Galileo Galilei dynamics reached the full.

It is difficult to say which elements, apart from the technological pressure, contributed to the development of the science of motion and the enlargement of mechanics, and in which measure they did it: natural philosophy, experimental activity and mathematics.

1. The concept of impetus recovered in the XIV century Europe, suggested the principle of inertia on (meta)physical basis. Galileo, however, made the principle an empirical law. The adoption of the principle of inertia led to the breakdown of the (Aristotelian) dogma for which a motion was always due to a force and speed was proportional to the applied force.
2. Even the other Aristotelian dogma, that of the speed of falling of heavy bodies proportional to their weight was abandoned. Someone, some mathematician – Stevin, as an example – dared to test the theory, and verified that it was false. Something that was probably obvious to most people, except philosophers.
3. Another key contribution was the introduction by Galileo of time as a physical quantity. Perhaps it was not quite Galileo to have first the idea. But it was he who first took note of the possibility of an accurate measure of

time, and developed the consequences. With the introduction of actual time, kinematics became dynamics. One could always imagine a motion evolving in an abstract time. Also the ancient Greek mathematicians did it, as for example Archimedes in the study of spiral. But they did so within geometry. By introducing the measurement of time geometry became charged with empirical significance and became dynamics. The law of Galileo for falling bodies, which somehow could find a counterpart in the kinematics of the Calculators of the XIV century, became with Galileo a law of nature.

4. An embryonic form of calculus allowed Galileo to pass from his law in term of constant increment of speed to the law of odd numbers for covered spaces, the only that could be tested by experiments.

So far it seems that mechanics evolved as a discipline of mathematical character, with no reference to experiments in the modern sense, that is the use of precise measurements, to verify theories. Even the role of philosophy of nature seems scarcely relevant because the concept needed to develop a mechanical theory could be derived from everyday observations without the need of the "abstruse" reasoning of philosophers.

The actual historical development, however, was a bit different. Galileo, differently from the traditional mixed mathematicians, had to intervene actively in the philosophy of Nature. Especially to free himself from preconceptions of natural philosophy.

When young professor in Pisa, in the 1590s, Galileo contrasted the Aristotelian theses on levity and gravity. He did it as a mathematician. In particular, he made recourse to thought experiments using the fundamental tool of mechanics, the lever, to argue against the existence of absolute levity. To establish principles as the law of inertia and that of falling bodies, he had to make recourse to reasoning in terms of cause and effect, as part of an essentially mechanistic philosophy, although not corpularistic. He referred only to material and efficient causes and denied the possibility of action at distance. He had to discuss the plausibility of motion of the Earth, for example, trying to provide a rational reconstruction to his law of relativity and confronting with Aristotelian philosophers who opposed his views.

Even the contrived experiment, carried out in laboratory, became an indispensable tool for the formulation of his laws, in particular those of the motion. Fundamental were the experiments of projection of heavy bodies moving on inclined planes that allowed him to choose from the two options, that of speed proportionally to the space of fall and that proportional to the elapsed time. Only after he had made his choice, he could carry on a purely "rational" exposition of the law of falling bodies.

After Galileo mechanics evolved inside the community of mathematicians. The evolution was due to a reflection on the subject, rather than to recourse to new experiments, which however had some role. The objective was to generalize Galileo's approach to cover situations more general than that of the motion of a mass point due to a constant gravity. Fundamental concepts were introduced, roughly corresponding to our force (Wallis, Newton) and energy (Huygens, Leibniz).

Protagonists of the evolution of the theory of mechanics, after Galileo, at the end of the XVII century, were Torricelli, Cavalieri, Descartes, Wallis, Huygens, Newton, Leibniz (and many others). Newton proposed a kind of mechanics that is still today an accepted model. With those of absolute time and space, transpiring from the background, in his mechanics fundamental concepts concerned mass and force. Neither was completely new however. The concept of mass could be found in a quite clear way in Baliani, Descartes and others, intended as quantity of matter, and distinct from weight, which was associated to mass both because the action of ethereal particles or attractive forces. The concept of force came from statics as cause of motion and equilibrium, in principle measured by the weight the force can rise.

Newton's mechanics, at least that exposed in the *Principia mathematicae philosophia naturalis* of 1687, had an axiomatic structure, based on three explicit (and many other implicit) principles. The three explicit principles, referred to as *leges sive axiomata*, are:

> Law I. Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
>
> Law II. The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
>
> Law III. To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts (Newton 1846, pp. 13-14).

Thousands of pages have been written on their logical status. This, for sure, means it is not easy to be grasped. One main doubt is if the laws are entirely *a priori* or derived from experience. Even though this second possibility is presently dominating, the experience called for is not that of the contrived experiments, but rather that of the common man, as was for Aristotle.

Newton attributed the first two laws to Galileo. There is historical motivation for this move, but for sure Newton was too generous. Especially for the second law. It could be considered a direct derivation of the Galilean law of motion. Indeed, if a constant cause (force) gives a motion with constant increase of speed – acceleration in modern term –, a variable cause (force) will give a motion with variable increase of speed; and it is not difficult to declare a proportionality between increase of velocity and cause (force). However, Newton's considered generic directions and causes, besides a mathematical apparatus Galileo did not possess.

Newton's *Principia* were considered by contemporaries a very clever text but not a revolutionary one, independently if its foundation was accepted or rejected. The text could indeed easily be framed into the tradition of mixed mathematics originated by Galileo, Wallis, Huygens. Only a modern perspective, and a particular attitude of the historian, could see in it something of revolutionary. The appreciation toward the *Principia* was due not so much – as is the case for modern scholars – to its foundation. It was not the terrestrial mechanics to be appreciated, but the celestial, with the proposed explanation of the planetary motion. Here experience came into play. It was represented by Kepler's laws from which Newton could derive (analytic phase) the universal law of gravitation. This quite simple law allowed in turn to deduce with the rule of Calculus the planetary motion (synthetic phase) and give a scientific foundation to the Copernican hypothesis.

Newton's mechanics had its limit in the restriction to the mass point free of any constraints. It was however adapted in the XVIII and XIX centuries to any situation: extended rigid bodies, constrained and deformable bodies.

2. Conclusions

This memoir dealt with the description of the evolution of old mathematics, actually mixed mathematics, in the Renaissance and Baroque era. This evolution depended on both external (pressure from society) and internal (development of mathematics and philosophy of Nature) causes. The theme dealt with in the memoir, the relation between science and mathematics, was the object of a tremendous amount of writings, especially in the 1950-1980s. Here it is proposed a quite new point of view that concentrates on professional mathematicians rather than on professional philosophers; assuming, however, that the former acted in fact as philosophers. Mathematicians – almost all of them having a quite good training in the philosophy of nature – were the only ones who could make homogeneous epistemology, natural philosophy and mathematics. This was not possible for professional philosophers, even when they were great mathematicians, as Descartes and Leibniz, that developed separately a natural philosophy and a (pure) mathematics.

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