# Searching for a response: Feynman’s work on the amplifier theory

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*Abstract*: Richard Feynman’s involvement in the Manhattan Project during the World War II is well known. He studied instruments and experimental devices, being directly involved, for instance, in the study of the “water boiler”, a small nuclear reactor designed for experimenting on fundamental properties of the chain reaction. In most of such experiments, the necessity emerged of feeding the output pulses of counters into amplifiers and various other circuitry, with the risk of introducing distortion at each step. To deal with this problem, Feynman designed a theoretical “reference amplifier” as an idealized device distorting the signal either at the low end or at the high one of its responsive range. In such a way, he was able to characterize the distortion by means of a benchmark relationship between phase and amplification for each frequency, thus providing a standard tool for comparing the working of real devices. In this contribution, we analyze Feynman’s work on amplifier response at Los Alamos, described in a technical report of 1946, as well as lectured on at the Cornell University in 1946-47 during his course on Mathematical Methods. Such a work later flowed in the Hughes lectures on Mathematical Methods in Physics and Engineering of 1970-71, where also causality properties were pointed out.

*Keywords*: Amplifier response, Causality property, Hughes lectures.

## 1. Introduction

The involvement of Richard P. Feynman in the Manhattan project (Galison 1998; Feynman 1985) is well known. There, he studied a number of different problems directly related or not to the making of the bomb. He was initially involved in studying instruments and experimental devices, such as for instance the “water boiler", a small nuclear reactor designed to experiment on fundamental properties of the chain reaction (Feynman 1946a). He also developed an integral theorem that allowed to evaluate the distribution of neutrons and active material from known distributions, in order to maximize the number of neutrons leading to a successful chain reaction (Feynman 1946b). Furthermore, he had to deal with numerical calculations concerning implosion plutonium bombs (Feynman 1945), rather than uranium ones, this last project being assigned to him by the theory division leader Hans Bethe, whom he would follow to Cornell University after the end of the war. Finally, the most important and difficult project concerned the "hydride bomb", which was supposed to work around a uranium hydride core, where the hydrogen atom in the hydride would favor the slowing down of neutrons originating the chain reaction, in this way consuming less $^{235}U$ than the ordinary metal bomb (Feynman, Welton 1947).

Summarizing, Feynman's work at Los Alamos was mainly concerned with technical and engineering issues. In most experiments, the basic aim was simply to count neutrons emerging from a given reaction, in order to estimate its efficiency, but neutron signals were usually so low that an amplifier was required to study them. The practical problem with feeding the output pulses of counters into amplifiers and various other circuitry was mainly the emergence of distortions at very high and very low frequencies. In order to solve such a problem, instead of studying the details of the different amplifiers employed in the different experiments, Feynman designed a theoretical "reference amplifier" distorting the signal either at the low or at the high end of its responsive range, thus providing a standard against which comparing the real devices. In particular, he succeeded in characterizing the distortion introduced by means of a benchmark relationship between phase and amplification of the signal for each frequency component. This interesting work is described in a technical report (Feynman 1946c), and later lectured on at the Cornell University in 1946-47, in a course on Mathematical Methods in Physics (Feynman 1946d). Here, the problem of deriving the response of an amplifier was worked out as an example within a section on the applications of contour integration methods and the residue theorem. Finally, the same amplifier problem was taken on in 1970-71, within a course on Mathematical Methods in Physics and Engineering, delivered at the Hughes Aircraft Company (Feynman 1971). Here such an issue was further developed with a focus on causality properties of the transfer function, succeeding even in deriving the Kramers-Kronig dispersion relations, whose standard framework (also considered by Feynman) is the application to the light refractive index (Lipson et al 2010).

In the present contribution, we deal with Feynman's theoretical reference amplifier, as inferred from the technical report of 1946, as well as from his Course on Mathematical Methods in Physics delivered in 1946-47 and finally from his 1970-71 Hughes lectures. The general theory developed by Feynman is highlighted in Section 2, while Section 3 is devoted to some theoretical issues he addressed later, concerning the causality properties of the amplifier and its link with the dispersion relations technique. Finally, in Section 4 some concluding remarks will be presented.

## 2. The amplifier response: general theory

During the development of the Manhattan project it was often necessary to amplify signals coming from neutron counters or ionization chambers. Usually, such signals are composed of different frequencies and, when entering an amplifier, amplification is not the same for all frequency components, thus introducing some distortion in the output signal. Phase shifts may as well develop for different frequency components, whose behavior as a function of the frequency can be assumed to be linear to a first approximation (Feynman 1946c). Under such an assumption Feynman was able to neglect the time delay and to “sum" a high pass and a low pass filter in order to get a theoretical “amplifier" with a behavior similar to that of a real device.

According to Feynman (1971), an amplifier can be regarded as a black box characterized by the fact that the output voltage $E_{\mathrm{OUT}}$ is related to the input voltage $E_{\mathrm{IN}}$ by a quantity $g$, a linear function known as the gain of the device:

$$E_{\mathrm{OUT}} = g\, E_{\mathrm{IN}}. \tag{1}$$

The amplifier was also assumed to be time-invariant: if at time $t$ the output signal $F(t)$ is obtained from the input one $f(t)$, this same sample signal in input at a later time $t+a$ will produce the same output. A good amplifier is flat over a large region of frequencies, that is amplification is nearly independent of frequency in this region, while, on the other hand, for very high and very low frequencies the amplification falls off rapidly. In particular, for high frequencies the amplification follows an inverse power law $(\omega_0/\omega)^k$, where $\omega_0$ is some characteristic frequency. Similarly, amplifiers with a low-frequency cutoff have amplification falling off as $(\omega/\omega_0)^k$. The high-frequency response affects the shape of a pulse, its rate of rise and the accuracy with which the pulse is followed, while the low-frequency counterpart determines the response over long times. Feynman performed a different analysis in these two situations, by considering two kinds of amplifiers: a first one having only a high-frequency cutoff while it is flat for low frequencies, and, conversely, a second one with a low-energy cutoff while passing with unit amplification all frequencies. Due to linearity, the effect of a real amplifier with both cutoffs can be obtained by letting the pulse pass first through a high-frequency cutoff amplifier and, then, through a second amplifier with a low-frequency cutoff only.

Feynman's peculiar approach was to study the response of the amplifier to a delta-function signal, and then constructing the response to a variety of differently shaped input signals by considering them as the superposition of a bunch of delta-functions, each at a given different time and weighted with a different amplitude (Feynman 1946c). Thus, for a pulse of general shape $f(t)$, written as the superposition of a very large number of delta pulses occurring at different times, the response of the amplifier is given by:

$$O(t) = \int_{-\infty}^{+\infty} f(t')\, R(t-t')\, \mathrm{d}t', \tag{2}$$

$R(t)$ being the response to the single $\delta(t)$ pulse, i.e. a Green’s function. On the basis of these assumptions, an input sine wave with constant frequency, $E_{\mathrm{IN}} = \exp(i\omega t)$, will produce an output with the same frequency, but amplified and phase shifted, $E_{\mathrm{OUT}} =$

$A(\omega)\exp(i\omega t)$, where $A(\omega)$ is the transfer function of the amplifier. The main focus of Feynman's analysis was just on this quantity.
In the general case of an input signal built of many frequencies, the output will depend on the amplitude of each component, so that integration over all frequencies is required in order to get the total output signal $E_{OUT}$, characterized by the Green's function:

$$R\left(t-t'\right)=\int_{-\infty}^{+\infty}\mathrm{e}^{i\omega(t-t')}A(\omega)\,\frac{\mathrm{d}\omega}{2\pi}\,. \tag{3}$$

Summarizing, Feynman deduced the features of an amplifier from its response to a pulse or to a sine wave of definite frequency. Given the general expression for $R(t)$, Feynman's analysis focused on the behavior of such a function for various choices of $A(\omega)$ (Feynman 1946c). He also briefly pointed out that a reliable $A(\omega)$ for a real amplifier has to satisfy given relations between frequency and phase shift in order to not allow output signals occurring before the introduction of an input signal, i.e. all singularities (poles and branch points) of $A(\omega)$ lie on the positive imaginary half of the complex plane. Such a mathematically-inspired method was inherited by the famous textbook by H.W. Bode (1945), originally written as a technical report for engineers, and subsequently turned into a book. Later, however, Feynman developed in more detail this issue in his Hughes lectures on Mathematical Methods in Physics and Engineering (Feynman 1971).

## 3. Causality and dispersion relations

A very interesting issue addressed by Feynman (1971) was the causality properties of an amplifier, namely the requirement that its response function $R(\tau) = 0$ for $\tau < 0$. Such an issue was pivotal in Feynman's approach to the amplifier, as apparent from the fact that it is mentioned in the Los Alamos report (Feynman 1946c).

In general, the concept of *strict* causality deals with the fact that no output can occur before the input. It can be conveniently expressed in different forms for different physical systems. For a homogeneous refractive medium, for instance, it can be read as no signal can be transmitted faster than the speed of light $c$. Causality reflects itself into dispersion relations, which are integral formulas relating a dispersive to an absorptive process: they are ubiquitous in physics, ranging from the theory of light dispersion in a dielectric medium to the scattering of nuclear particles (Nussenzveig 1972), as well as the electrical network theory (Bode 1945). A dispersion relation is expected to hold in any theory where the output function of time is a linear functional of an input function, the interaction being time-independent, and where the output function cannot manifest before the application of the input one. The requirement that no response occurs until the application of an input signal can be expressed as (Lipson *et al.* 2010):

$$\int_{-\infty}^{0} R(\tau)\, \mathrm{e}^{-i\omega'\tau} \mathrm{d}\tau = 0, \tag{4}$$

which, upon substituting Eq. (3) and making some manipulations, becomes:

$$\int_{-\infty}^{+\infty} \frac{A(\omega')}{\omega' - \omega - i\epsilon} \frac{\mathrm{d}\omega'}{2\pi} = 0\,. \tag{5}$$

Thus, the causality condition can be translated by requiring that $A(\omega)$ has no singularities below the real axis in the plane of complex frequencies. A given function exhibits a pole for a given complex frequency $\omega = \omega_R + i\omega_I$ when a resonance is present: by approaching the resonant frequency, the oscillation amplitude becomes infinite for a driving force with finite amplitude. In this way, the causality principle suggests that the only way a physical system can achieve an infinite amplitude is as a result of its memory of an infinite driving force at some earlier time.

Finally, when dealing with the properties of the transfer function $A(\omega)$, Feynman introduced the concept of dispersion relations in his discussion. Indeed they can be extracted from the causality condition, Eq. (5), for the complex function $A(\omega) = A_R(\omega) + \mathrm{i}\, A_I(\omega)$:

$$\int_{-\infty}^{+\infty} A_R(\omega')\, \mathrm{p.v.}\left(\frac{1}{\omega' - \omega}\right) \frac{\mathrm{d}\omega'}{\pi} \;=\; A_I(\omega), \tag{6}$$

$$-\int_{-\infty}^{+\infty} A_I(\omega')\, \mathrm{p.v.}\left(\frac{1}{\omega' - \omega}\right) \frac{\mathrm{d}\omega'}{\pi} \;=\; A_R(\omega). \tag{7}$$

In optics, as Feynman noted, the function $A(\omega)$ represents the complex refractive index of light: its imaginary part describes light absorption by a medium, while the real part gives the frequency-dependent refractive index $n$ (a phenomenon known as chromatic aberration).

## 4. Conclusions

In this contribution, we have reported on an analysis of Feynman's work on amplifier response performed at Los Alamos and described in a technical report of 1946, as well as lectured on at the Cornell University in 1946-47 during his course on Mathematical Methods. Such a work later flowed in the Hughes lectures on Mathematical Methods in Physics and Engineering of 1970-71, where he also discussed causality properties and their equivalence to dispersion relations. Such a work grew out during his involvement in the Manhattan Project, where experiments required to feed the output pulses of counters into amplifiers or several other circuitries, with the risk of introducing distortion at each step. These issues were addressed by Feynman through the development of the idea of a theoretical "reference amplifier" able to provide a useful standard in practical comparison with real devices. He built up a general theory, relying strongly on the response function $R(t)$ of that amplifier (assumed to be linear). In particular he was able to find the basic features of an amplifier from its response to a pulse or to a sine wave of definite frequency. The main properties of the response function were explicitly worked out, starting from the key role played by the causality issue, i.e. certain relations between frequency and phase shift that a real amplifier has to satisfy in order not to allow output signals to appear before input ones. Finally, Feynman pointed out the equivalence between causality property and dispersion relations to be satisfied by the response function, probably inspired by similar issues in different physical contexts.

From our analysis, once more one can see the original approach of Feynman to scientific problems at work in a quite unusual field of application.

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